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The Macroeconomics of Model T

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Abstract

We study a model of growth and mass production. Firms undertake either product innovations that introduce new luxury goods for the rich; or process innovations that transform existing luxuries into mass products for the poor. A prototypical example for such a product cycle is the automobile. Initially, an exclusive product for the very rich, the automobile became affordable to the middle class after the introduction of Ford's *Model T*, "the car that put America on wheels". We present a model of non-homothetic preferences, in which the rich consume a wide range of exclusive high-quality products and the poor a more narrow range of low-quality mass products. In this framework, inequality affects the composition of R&D through price and market size effects. The inequality-growth relationship depends on how mass production affects productivity; and on the particular dimension of inequality (income gaps versus income concentration). Our model is sufficiently tractable to incorporate learning-by-doing, oligopolistic market structures, and different sources of knowledge spillovers.

JEL classification: O15, O30, D30, D40

Keywords: inequality, technical change, growth, mass production, product innovations, process innovations.

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"Consumer goods inventions that cut both cost and quality but reduce the former more than the latter, such as the Model T, have historically been an important means for transforming the luxuries of the rich into the conveniences of the poor."

Jacob Schmookler, *Invention and Economic Growth* (1966)

1 Introduction

This paper develops a model of endogenous growth and mass production based on product and process innovations. Product innovations introduce new goods which are affordable only to the rich. Process innovations implement mass production technologies and introduce low-quality versions of existing products, making the good affordable to the poor. As emphasized by Schmookler (1966), the joint introduction of mass production technologies and lower quality mass products have historically been important to transform the luxuries of the rich into the conveniences of the poor.

The automobile, one of the most important durable goods in modern industrial societies, provides a prototypical example for such an innovation cycle. In the United States, the history of the commercial automobile production started with Charles and Frank Duryea who founded the Duryea Motor Wagon Company in 1893, the first American automobile manufacturing company followed by Oldsmobile and Cadillac in 1902 and 1903. At the time, the automobile was a luxury good consumed only by very rich households. Things started to change in 1908, when Ford introduced the *Model T,* the car that "put America on wheels". The concept was the use of assembly lines and interchangeable parts to produce a low-cost, low-quality car affordable to the middle class. Model T became a huge success and initiated the takeoff in car ownership in the U.S. Between 1908 and 1927 more than 15 million units of Model T were manufactured. The introduction of Model T contributed crucially to the fast diffusion of the automobile in the U.S.[1] Consumer goods inventions that turn luxury goods into low-quality, low-cost mass consumption goods are not confined to the auto industry. In fact, many other consumer durables, such as the refrigerator, the radio, the TV, and more recently, the computer, show a qualitatively similar product cycle.

In our model income inequality affects both the incentive to invent new products and the incentive to introduce low-cost, low-quality mass products. There are rich and poor households who have the same preferences over differentiated consumer goods. Consumer goods are indivisible. Households decide whether or not to consume a certain good and, if yes, whether to

[1] Encyclopaedia Britannica. For a more detailed description of the evolution of mass production in the U.S. in general, and the role of the Ford Motor Company and the car market in particular, see Hounshell (1984).

consume it in high or low quality. This generates a tractable framework in which we can study how income distribution affects firms' market demand functions and innovation incentives.[2] This framework also captures in a stylized way the (empirically relevant) idea that rich households consume high qualities and a wide range of goods, whereas poor households consume a more narrow range in standardized low-quality goods.

To keep things simple, we consider a two-class society. In such a set-up we ask: How does inequality affect mass production and long-run growth? The answer depends on two crucial issues. First, it is important whether higher inequality is due to a larger income gap or to higher income concentration.[3] A larger income *gap* increases the prices the rich are willing to pay for new products. This directs R&D incentives towards product innovations and reduces the incentive for process innovations. Higher income *concentration* also raises prices and incentives for product R&D. However, higher concentration has an additional effect on the allocation of resources: there are fewer consumers purchasing high-cost/high-quality products and more consumers purchasing low-cost/low-quality products. The *second* crucial issue concerns the resource-intensity of mass production. There are two opposing effects. On the one hand, additional mass production is resource-consuming because of a *demand-effect*: more mass production requires the set-up of additional production facilities and additional production workers. On the other hand, there is a *productivity-effect*: a higher prevalence of mass production contributes to technical progress when process innovations increase the economy's knowledge stock. We show that, when the productivity effect dominates, larger income gaps reduce growth while the effect of higher income concentration is ambiguous. In contrast, when the demand effect dominates, larger gaps and higher concentration both increase growth.

We extend our basis framework in various directions. Our first extension considers *competition by outsiders*. The basic model assumes that both the high- and the low-quality version of a particular product are supplied by the same firm. In reality, however, the different qualities are

[2]Non-homotheticities arising from indivisible goods focus on the *extensive* margin of consumption choices. This is orthogonal to the (homothetic) CES framework which emphasizes the *intensive* margin. With CES preferences, poor households consume the same menu of goods as rich households though in lower quantities. Consumption indivisibilities have been analyzed, for instance, by Murphy, Shleifer and Vishny (1989) in the context of industrialization and economic development and by Matsuyama (2000) in the context of Ricardian trade.

[3]By a larger *income gap* we mean a larger income distance of rich and poor households, holding population shares of the two goups constant. By a higher *income concentration* we mean a larger share of poor households with same income and a smaller share of rich households with a higher income. (Larger gaps and higher concentration refer to mean-preserving spreads. The associated Lorenz-curve is piecewise linear. The income gap is measured by the slope of the first segment; income concentration is measured by the share of the poor, see section 3.1 below).

often supplied by different firms. We therefore study an alternative set-up in which the Model T version of an existing product is invented and produced by an outsider. The incumbent high-quality producer and the low-quality producing entrant engage in Bertrand competition. In such a setting, the product innovator has a strictly weaker incentive than outsiders to implement the mass production technology. Hence, in equilibrium, all process R&D is performed by outsiders. Moreover, compared to the monopoly case, in the oligopolistic equilibrium, more resources are invested in the process R&D. In sum, allowing for competition by entrants affects growth and mass production by changing the innovation mix in favor of process R&D. When the productivity effect of mass production exceeds the demand effect, entry competition increases growth and vice versa.

Our second extension allows for deterministic *product cycles*. Our basic framework assumes perfect symmetry across goods. It determines the percentage of sectors implementing mass production technologies, but leaves the life cycle of an individual product indeterminate. We show two extensions that generate deterministic cycles. We first introduce cost-asymmetries through *learning-by-doing* at the firm level. In that case, the "oldest" exclusive producer (with most production experience) has the highest incentive to introduce the next mass product. Introducing learning-by-doing also affects the inequality-growth relationship. As a more egalitarian society learns more quickly (because demand is concentrated on a more narrow range of sectors), the learning-by-doing effect makes inequality more harmful (or less beneficial) for growth. Changes in inequality that increase mass production generate high learning effects and may increase growth even in the absence of a knowledge spillovera effect (i.e. when spillovers arise entirely from product innovations). An alternative way to generate deterministic product cycles are *hierarchic preferences*. When goods can be ordered by priority in consumption, process and product innovations follow the sequence determined by the consumption hierarchy leading to a cycle where firms producing the high-quality good with highest priority have the highest incentive to implement the mass production technology. In such a framework, our main conclusions concerning the inequality-growth relationship remain qualitatively unchanged.

Our third extension accounts for *quality upgrading*. The equilibrium of our model features a growth path characterized by an expanding variety of consumer goods supplied in two constant qualities. In reality, however, the quality of most products is constantly increasing, often featuring a situation where the quality of the mass product is substantially higher than the quality of the original exclusive product. We show that our framework lends itself nicely to incorporating rising product qualities. When R&D activities do not only generate knowledge spillovers in research but also in producing quality, it is straightforward to show that the quality of the current mass product may well exceed the quality of the original exclusive products. Alternatively, we consider the case where quality upgrading is the endogenous by-product of production experience.

Our analysis extends the existing literature in at least three dimensions. *First*, our paper highlights the distinct role of product and process innovations in models with non-homothetic preferences. In standard R&D based growth models (Romer 1990, Aghion and Howitt, 1992, Grossman and Helpman 1991, etc.) product and process innovations are mathematically very similar (Acemoglu, 2009). In particular, a model of product innovations (that introduces new or better consumer goods) can typically be re-interpreted as a model of process innovations (new or better intermediate inputs) without generating any major differences in basic insights. In contrast, under non-homothetic preferences, the distinction between product and process innovations becomes vital. Process and product innovations affect rich and poor consumers in a different way; and the innovation mix itself is, in a non-trivial way, affected by the distribution of income. As a consequence, the welfare implications of growth policies depend on the particular source that drives economic growth.

Second, our paper is related to the literature on directed technical change (Acemoglu, 1998 and 2002, Acemoglu and Zilibotti, 2001, and others). This literature analyzes the forces that generate biases in technical change towards one particular production factor. Similar to our paper, directed technical change models emphasize the tension between price and market size effects that determine the allocation of productive resources to alternative R&D activities. Directed technical change models put emphasis on the relative demand for production factors, i.e. the supply/cost side of the economy. In contrast, our model focuses on the relative demand for consumption goods, i.e. exclusive goods versus mass products. The demand channel highlights the distribution of income across households as a mechanism determining the level and composition of R&D. This mechanism that is absent in directed technical change models.

Third, our paper contributes to the literature that studies the impact of income inequality on technical progress via the composition of aggregate consumer demand. Matsuyama (2002) demonstrates the virtuous cycle between learning-by-doing and a large middle class, enabling the "flying-geese" pattern in which new products take off one after another, due to rising productivity and income. Foellmi and Zweimüller (2006) focus on product inventions and innovators' price setting power in the presence of a wealthy upper class. In such a set-up it turns out that inequality is unambiguously beneficial for growth. The present paper can be viewed as a synthesis of these classes of models. Our analysis highlights the conditions under which an unequal society suffers from lack of process innovations (and/or learning-by-doing) and from a small range of mass markets. Our analysis also makes precise the conditions under which such a society benefits from large mark-ups and high incentives to open up completely new product lines.[4]

The paper is organized as follows: Section 2 analyzes empirical and historical evidence motivating the key assumptions and mechanisms of our model. Section 3 introduces the formal framework, section 4 presents the solution of the balanced growth equilibrium, and section 5 discusses the relationship between inequality and growth. In section 6 we relax several of our assumptions and introduce alternative specifications of market structure, distribution, preferences and technology. We conclude with a summary and potential directions for future research.

2 Motivating evidence

Casual observations and empirical evidence suggest that there is a strong impact of income on the number of varieties purchased by households, which is at odds with homothetic preferences.[5] Figure illustrates this point by exhibiting the shares of ownership of various consumer durables of urban Chinese households (National Bureau of Statistics of China). At any given point in time, most types of consumer durables are only consumed by a fraction of the households. The figure also shows that levels of penetration rise over time. This is what Matsuyama (2002) calls the "Flying Geese pattern", in which a series of products takes off one after another, following an increase in productivity and income. This gradual increase in penetration levels was first emphasized by Katona (1964) who observed that the mass consumption society is the last stage of a process in which former luxury goods, consumed only by a few, privileged households, have been transformed into necessities for most households (i.e. mass consumption goods). Many products such as cars, radios, television sets, washing machines, refrigerators, vacuum cleaners and, more recently, computers have gone through such product cycles in the developed world, and are presently going through similar cycles in developing countries. Besides plain income effects, key elements of such product cycles are cost-saving process innovations. After a product has been invented, initial manufacturing costs are usually quite high, and sales volumes low as the good can only be afforded by a few rich households. The takeoff and subsequent

[4]Murphy, Shleifer, and Vishny (1989) study the role of income distribution on technology adoption in a static context. Falkinger (1994) develops a model where inequality affects technical progress via aggregate output of consumer goods. The effect of inequality on technical progress in quality ladder models is explored in Li (2003) and Zweimüller and Brunner (2005).

[5]Jackson (1984) finds that the richest income class consumed twice as many different goods as the poorest class, using micro data from the Consumer Expenditure Survey of the Bureau of Labor Statistics. Falkinger and Zweimueller (1996) generate similar results using aggregate cross-country data from the International Comparison Project of the UN on per-capita expenditure levels on ninety-one different consumption categories.

proliferation of the product is often ignited and enabled by a series of process innovations that reduce manufacturing costs significantly.[6]

FIGURE 1

As mentioned above, one of the most famous historical examples for such an innovation pattern is the Ford Model T. One major reason behind the huge success story of Model T were Ford's innovations, including assembly line production instead of individual hand crafting, as well as the concept of paying the workers a wage proportionate to the cost of the car, so that they would provide a ready made market. Both innovations led to a huge increase in productivity. In total, Ford manufactured more than 15 million Model T's from 1908 to 1927, which contributed critically to the fast diffusion of the automobile. Figure shows automobile and truck registrations in the U.S. from 1900 to 1970. The number of car registrations took off in the period of the Model T, and reached 23 million in 1927. Whereas 1% of households in the U.S. owned a car in 1908, the hour of birth of the Model T, penetration reached 50% in 1924.[7]

FIGURE 2

The product cycle that led to the Model T is not specific to the U.S. but can be observed in other parts of the world. Most of the large European economies had their own Model T which brought the car to the people. In Germany, a "people's car" – Volkswagen ("Beetle") – was initially introduced in the 1930s (and fostered by the Nazi regime). Austin 7 (1922), Fiat (1936) and Citroën (1949)[8] brought the car to the people of the UK, Italy and France, respectively. In rich countries, the introduction of mass-produced cars was an important step in the history of manufacturing. What has been historically important for rich countries is starting to become relevant in poorer countries today. In Asia for example, Tata has recently announced to produce the world's cheapest car, mainly for the Indian market.

The auto industry is an example for the types of innovation and product cycles that our

[6]Our analysis highlights the relevance of major product and process innovations that create new product lines and subsequent mass consumption goods. Notice that in reality both mass consumption goods and luxury goods are continuously improved in quality. While this is clearly of high relevance in practice, we abstract from continuous quality improvements in our framework.

[7]See Model T Facts on media.ford.com, Encyclopaedia Britannica, and Bowden and Offer (1994) for penetration levels.

[8]Citroën director Pierre-Jules Boulanger's early design brief for the 2CV supposedly asked for "a vehicle capable of transporting two peasants in boots, 100 pounds of potatoes or a barrel of wine, at a maximum speed of 40 mph, [...] Its price should be well below the one of our Traction Avant and, finally, its appearance is of little importance." (Translation, Technologie SCEREN - CNDP no. 138, 2005)

model aims to capture. While it provided the classical example, there are many other goods that experienced very similar patterns of innovation and market expansion. Two centuries after artificial refrigeration was pioneered by Dr. William Cullen, a GE home *refrigerator* cost around 700$ in 1922, compared to 450$ for a 1922 Ford Model T. Penetration barely reached 1% in the U.S. in 1925. The introduction of freon expanded the refrigerator market during the 1930s, with penetration reaching 50% by 1938. Refrigerators went into mass production after WWII, and by the year 1948 75% of all households owned a fridge.[9] The history of *television* started with first experimental transmissions made by Charles Jenkins in 1923. Television usage in the U.S. exploded after WWII. Having reached a penetration of 1% in 1948, it only took 5 years to reach 50%, and 2 more years to reach 75%. The rapid diffusion was enabled by the lifting of the manufacturing freeze, war-related technological advances, the expansion of the television networks, the drop in television prices enabled by mass production and additional disposable income.[10] A very similar evolution can be traced for *computers*. Spurred by calculation requirements for ballistics and decryption during WWII, the first electronic digital computers were developed between 1940-1945. Developments of the microprocessor led to the proliferation of the personal computer after about 1975. Mass market pre-assembled computers allowed a wider range of people to use computers, and penetration reached 1% in the U.S. around 1980. Component prices continued to fall since then, leading to continuous price declines. Penetration reached 50% around 2000 and increased further.[11]

These examples demonstrate that process innovations and mass consumption markets are intertwined: Process innovations reducing manufacturing costs are crucial elements for tapping and proliferating mass consumption markets. Mass production, in turn, facilitates process innovation by increasing learning-by-doing and specialization benefits. Higher inequality raises the purchasing power of rich households, increasing demand for variety and product innovation. A more egalitarian society, on the other hand, raises the number of mass consumption markets and thus incentives for process innovation. Comparing the experience of Japan and the U.S. over the last decades provides suggestive evidence: Income concentration in Japan has remained relatively low after WWII in contrast to the U.S. (Moriguchi and Saez, 2005). During the same period of time, Japan has made itself a name as country of lean production and just-in-time management, i.e. process innovation. A recent study by Nagaoka and Walsh (2009), using data from the RIETI-Georgia Tech inventor survey, indeed shows that R&D in Japan is more biased

[9] Association of Home Appliance Manufacturers, "The Story of the Refrigerator;" Bowden and Offer (1994)

[10] Steven Schoenherr, "History of Television," History Server of University of San Diego; Bowden and Offer (1994)

[11] Jeffrey Shallit, "A Very Brief History of Computer Science," University of Waterloo; W. Warner, "Great Moments in Microprocessor History," Technical Library IBM; "Computer Use and Ownership," U.S. Census.

to process innovation, in contrast to the U.S. where it is more directed to product innovation.

3 The model

3.1 The distribution of endowments

We assume there are L households that inelastically supply L units of labor. $\beta L < L$ households are poor (indexed by P) and $(1-\beta)L$ are rich (indexed by R). Income differences arise from two sources. First, households are unequally endowed with units of labor. A poor household is endowed with $\ell_P = \theta_\ell < 1$ labor units, and the labor endowment of a rich household is $\ell_R = (1-\beta\theta_\ell)/(1-\beta) \geq 1$.[12] The parameters β and θ_ℓ fully characterize the distribution of labor endowments. Figure 3 shows the corresponding Lorenz-curve. It is piecewise linear with slope θ_ℓ for population shares between 0 and β; and slope $(1-\beta\theta_\ell)/(1-\beta)$ for population shares between β and 1. Notice that common measures of inequality (such as the Gini coefficient and the coefficient of variation) indicate an increase in inequality when θ_ℓ falls and/or β rises. It is assumed that the distribution of labor endowments is constant over time.

FIGURE 3

The second source of income differences is due to inequality in wealth, based on ownership in monopolistic firms. We denote by $v(t)$ the per-capita value of these firms at date t and assume that a poor household owns wealth $v_P(t) = \theta_v(t)v(t)$ and a rich household owns wealth $v_R(t) = [(1-\beta\theta_v(t))/(1-\beta)]\,v(t)$ where $\theta_v(t) < 1$ and $(1-\beta\theta_v(t))/(1-\beta) \geq 1$. In analogy to the labor endowment distribution, the distribution of wealth is determined by β and $\theta_v(t)$. Unlike the labor endowment distribution, the wealth distribution can change over time since $v_P(t)$ and $v_R(t)$ are endogenously determined by households' savings decisions. In sections 4 to 6 below we will study balanced growth paths. Along such paths, all households have the same savings rates and the wealth distribution is stationary, $\theta_v(t) = \theta_v$ for all t. When we analyze balanced growth paths below we will assume $\theta_\ell = \theta_v = \theta$. While this is clearly a rather special case, it keeps the analysis simple and transparent. Allowing labor endowment and wealth distributions to differ does not change the results in any economically relevant way. For instance, in comparing steady states, it does not make a difference whether the resulting incomes differences arise due to an unequal labor endowment distribution, due to an unequal wealth distribution, or both. What matters is inequality in total lifetime incomes. In the online Appendix, where transitional dynamics are studied, we have to account for the fact that households' savings rates need no

[12] Since the average labor endowment per household is unity we must have $\beta\ell_P + (1-\beta)\ell_R = 1$. Setting $\ell_P = \theta_\ell$ we get $\ell_R = (1-\beta\theta_\ell)/(1-\beta)$.

longer be equal in the transition to a new steady state. As the wealth distribution changes over time we have to abandon the assumption $\theta_\ell = \theta_v = \theta$ and make the time-dependence of $\theta_v(t)$ explicit.

We will refer to a lower value of θ as an increase in *income gaps* and to a higher value of β as an increase in *income concentration* (there are more poor households with the same endowment and fewer rich households with a higher endowment).

3.2 Preferences and consumer choices

Households have an infinite horizon and choose consumption both within and across periods to maximize lifetime utility. At a given point in time, a household chooses consumption from the continuum of $N(t)$ goods. Among the $N(t)$ firms that exist at date t there are $N(t) - M(t)$ firms that made a product innovation but have not yet made a process innovation (exclusive producers) and $M(t)$ firms that have made both the product and the process innovation (mass producers). This means $M(t)$ goods are supplied both in high and low quality and $N(t) - M(t)$ goods are supplied in high quality only. In general, the prices may vary both across goods and across qualities and may change over time. We denote the price of good j and quality q at date t by $p(j, q, t)$.

The crucial assumption adopted here is that goods are indivisible. More precisely, the household has to decide whether or not to consume good j, and if yes, whether to consume it in high or low quality. There are three outcomes: either a household consumes (i) one unit in high quality, (ii) one unit in low quality, or (iii) does not consume at all. It turns out that such a discrete specification of preferences is a simple and tractable way to introduce non-homotheticities and to allow for a situation where rich households do not only consume a broader menu of goods but also consume the purchased goods in higher quality. Denote by $x_i(j, t)$ an indicator function that takes value 1 if household i consumes good j at date t, and takes value 0 if not. Similarly, denote by $q_i(j, t)$ the chosen quality level which can take only one of the two values $\{q_h, q_l\}$. The household's objective function is given by

$$U_i(\tau) = \int_\tau^\infty \frac{1}{1-\sigma} \left[\int_0^{N(t)} x_i(j,t) q_i(j,t) dj \right]^{1-\sigma} e^{-\rho(t-\tau)} \, dt,$$

where ρ is the rate of time preference, and σ parametrizes the willingness to shift consumption across time. The term in brackets can be interpreted as an instantaneous consumption aggregator which, for later use, we denote by $c_i(t) \equiv \int_0^{N(t)} x_i(j,t) q_i(j,t) dj$. The consumer chooses the time paths of $x_i(j, t)$ and $q_i(j, t)$ so as to maximize the above lifetime utility subject to the lifetime budget constraint

$$\int_{\tau}^{\infty}\left[\int_{0}^{N(t)} p(j,q_i,t)x_i(j,t)dj\right]e^{-R(t,\tau)}dt \leq \int_{\tau}^{\infty} \ell_i w(t)e^{-R(t,\tau)}dt + v_i(\tau),$$

where $R(t,\tau)=\int_{\tau}^{t} r(s)ds$ is the cumulative discount factor between dates τ and t, $r(t)$ is the interest rate, ℓ_i is the (time-invariant) labor endowment of household i, and $v_i(\tau)$ is the initial wealth level owned by the household.

The first-order conditions for the discrete consumption choice of good j are given by

$$\{x_i(j,t),q_i(j,t)\}=\begin{cases}\{1,q_h\} & \text{if} \quad q_h\mu_i(t)-p(j,q_h,t)\geq \max\left[0,q_l\mu_i(t)-p(j,q_l,t)\right],\\ \{1,q_l\} & \text{if} \quad q_l\mu_i(t)-p(j,q_l,t)\geq \max\left[0,q_h\mu_i(t)-p(j,q_h,t)\right],\\ \{0,\cdot\} & \text{otherwise,}\end{cases} \tag{1}$$

where

$$\mu_i(t)=c_i(t)^{-\sigma}/\lambda_i(t)$$

is household i's willingness to pay per unit of quality and $\lambda_i(t)$ the marginal utility of wealth at date t (the current-value multiplier). These first order conditions are very intuitive. The condition in the first line of (1) says that good j will be consumed in high quality if the consumer's willingness to pay for the high quality $q_h\mu_i(t)$ is sufficiently larger than its price $p(j,q_h,t)$ so that both alternatives (purchasing not at all and purchasing the low quality) lead to a worse outcome. In other words, there needs to be a utility gain and it needs to be larger than the utility gain from purchasing the low quality. Similarly, the consumer will purchase the low quality if there is a utility gain that is larger than when purchasing the high quality. Otherwise, the household does not consume good j at all.

3.3 Technology and prices

Labor is the only production factor, the labor market is competitive and the market clearing wage is denoted by $w(t)$. Production activities are undertaken in monopolistic firms that supply differentiated products and operate with an increasing returns-to-scale technology. The creation of a firm requires a *product innovation*, i.e. an investment of $\tilde{F}(t)$ units of labor that yields the blueprint for a completely new product (e.g. the automobile). Once such a product innovation has been made, the innovating firm obtains a patent of infinite length granting the exclusive right to market this product. Such an innovated producted is a luxury good in the sense that it is costly to produce and may be affordable initially only to the rich. We assume a new product has quality q_h and requires a (high) labor input $\tilde{a}_h(t)$ per unit of output. After a successful product innovation, the firm has the option to undertake a *process innovation* that cuts both the quality of the product and its production cost. More precisely, we assume that after a further investment of $\tilde{G}(t)$ labor units, the product can also be supplied in lower quality $q_l < q_h$. We assume that this further investment is also associated with the implementation of a more productive technology (a new "process") that implies that the low quality good can be produced with a lower labor input $\tilde{a}_l(t) < \tilde{a}_h(t)$. We assume that the process innovation lets the new (low-quality) version of the product be produced at a higher quality-cost ratio, $q_l/\tilde{a}_l(t) > q_h/\tilde{a}_h(t)$. This captures Schmookler's idea that mass consumer good inventions cut both costs and quality but the former more than the latter.[13]

Firms make their pricing decisions on the basis of market demand functions that derive from households' optimal consumption choices given by the conditions in (1). Notice that the willingness to pay for quality $k \in \{l, h\}$ is always larger for a rich household than for a poor household, $q_k\mu_R > q_k\mu_P$. For simplicity, we omit time indices in the remainder of this section. In what follows we will refer to firms that have incurred both the product and the process innovation as *mass producers*. Firms that have made only the product but not the process innovation will be called *exclusive producers*. The term "exclusive" is suggestive in the sense that it refers to both a high, exclusive quality and to a situation where firms "exclude" the poor from consumption by setting prices that only rich but not poor households can afford.[14]

An *exclusive producer* can supply the product only in high quality. When the firm charges a price below (or equal to) $q_h\mu_P$ both rich and poor households will purchase the good and market demand is L. When the price is above $q_h\mu_P$ but below (or equal to) $q_h\mu_R$ only rich households purchase the good and market demand is $(1-\beta)L$. When the price is larger than $q_h\mu_R$ not even the rich are willing to purchase and market demand is zero. The exclusive producer has essentially two options: (i) set price $q_h\mu_R$ and sell to rich households only; or (ii) set price $q_h\mu_P$ and sell to the whole customer base.

A *mass producer* can supply the good both in high and low quality. The mass producer

[13]Note that we abstract from continuous quality improvements of existing goods which are important features of reality. The model could be easily adapted to include exogenous quality improvements. If q_h and q_l increased at an exogenous rate, all features of our model would remain the same. We will also touch upon quality improvements when discussing learning-by-doing in Section 6. Furthermore, both high- and low-quality versions of a variety are produced by one firm as a first approximation. In reality, process innovations are often undertaken by competitors to enter an existing product line. Hence, one could extend the present model to a duopolist setting to study the competitive effects of the process innovator on the original product inventor.

[14]Note that the way we use the terms "exclusive producers" and "mass producers" refers to access to technology rather than to quantity of production. A priori, it is possible that a mass producer is better off by selling only to the rich and an exclusive producer is better off by selling to both the rich and the poor; but those "strange" outcomes can be ruled out along a balanced growth path.

has essentially five options: (i) supply only the low quality at price $q_l\mu_P$ to all households; (ii) supply the low quality at price $q_l\mu_R$ only to rich households; (iii) supply the high quality at price $q_h\mu_R$ only to rich households; or (iv) supply the high quality at price $q_h\mu_P$ to all households. Actually, the mass producer has a fifth option: (v) set price $q_l\mu_P$ for the low quality and sell it to poor households and set price $q_l\mu_P + (q_h - q_l)\,\mu_R$ for the high quality and sell it to rich households.[15]

In the present context, the most interesting case is a *separating equilibrium.* In that case all exclusive producers choose option (i) and all mass producers choose option (v). In that case, rich households purchase all products supplied by exclusive and mass producer (all of them in high quality); and poor households consume all of products supplied by mass producers (all of them in low quality).

3.4 R&D, resources and technical progress

It is assumed that entry into the *R&D sector* is free, so innovators make zero profits in equilibrium. Inventing a new good and setting up a new exclusive firm is attractive as long as the value of this product innovation (the present value of future cash flows) does not fall short of the initial R&D cost. Initial R&D costs are $w(t)\tilde{F}(t)$. The present value of a new innovation depends on whether and, if so, when the firm implements the mass production technology. The process innovation costs are $w(t)\tilde{G}(t)$. Denote by $\Delta(j)$ the duration between the product innovation and the process innovation (the "age" at which the firm implements the mass production technology); and by $\pi_e(j,t)$ and $\pi_m(j,t)$ the profits before and after implementing mass production, respectively. The value of a firm that introduces a new product at date τ is then given by

$$V(j,\tau) = \max_{\Delta(j,\tau)} \left[\int_{\tau}^{\tau+\Delta(j)} \pi_e(j,t)e^{-R(t,\tau)}dt + \int_{\tau+\Delta(j)}^{\infty} \pi_m(j,t)e^{-R(t,\tau)}dt - w(t)\tilde{G}(t)e^{-R(\tau+\Delta(j),\tau)} \right],$$

With free entry into the R&D sector, the general equilibrium leaves no profit opportunities unexploited. Hence the value of a product innovation cannot exceed the initial R&D cost $V(j,t) \le w(t)\tilde{F}(t)$.

The *labor market* is competitive so the economy's resources will be fully utilized at all times. Aggregate labor supply is fixed to L. Aggregate labor demand comes from the R&D sector

[15]Notice that under this fifth option the firm cannot fully exploit the willingness to pay of rich consumers since they can switch to the low quality. To attract the rich households as customers for the high quality, the firm needs to set a price that is not larger than the price that makes a rich household indifferent between consuming the low quality and consuming the high quality. From (1) it is straightforward to verify that, when the low quality has price $q_l\mu_P$, the highest price that induces the rich to purchase the high rather than the low quality is $q_l\mu_P + (q_h - q_l)\,\mu_R$.

and the production sector which produces (high- and low-quality) output. In the R&D sector, $\dot{N}(t)\tilde{F}(t)$ units of labor are engaged in designing entirely new products, and $\dot{M}(t)\tilde{G}(t)$ units of labor are used to implement new mass production technologies. In the production sector $Y_h(t)\tilde{a}_h(t)$ and $Y_l(t)\tilde{a}_l(t)$ units of labor are employed to produce high-quality and low-quality output denoted by $Y_h(t)$ and $Y_l(t)$, respectively. The resource constraint of the economy can be written as

$$Y_h(t)\tilde{a}_h(t) + Y_l(t)\tilde{a}_l(t) + \dot{N}(t)\tilde{F}(t) + \dot{M}(t)\tilde{G}(t) \leq L. \tag{2}$$

Technical progress is driven by product and process innovations. Sustained growth is enabled by knowledge spillovers from past research activities on current productivity levels. It is assumed that labor requirements in the various activities are inversely related to an aggregate stock of knowledge $A(t)$ such that $\tilde{F}(t) = F/A(t)$, $\tilde{a}_h(t) = a_h/A(t)$, $\tilde{G}(t) = G/A(t)$, and $\tilde{a}_l(t) = a_l/A(t)$ where F, G, a_h, and a_l are exogenous, positive constants. We use the wage per efficiency unit $w(t)/A(t)$ as the numéraire, which implies that nominal labor costs in the various activities are $w(t)\tilde{F}(t) = F$, $w(t)\tilde{a}_h(t) = a_h$, $w(t)\tilde{G}(t) = G$, and $w(t)\tilde{a}_l(t) = a_l$ which are constant over time. To generate a balanced growth path, the stock of knowledge $A(t)$ needs to be a linearly homogeneous function of the range of product varieties $N(t)$ and the subset of varieties that underwent process innovations $M(t)$. For analytical convenience, we assume that $A(t)$ is linked to past product and process innovations via the CES-function

$$A(t) = \left[\psi N(t)^{\gamma} + (1-\psi)M(t)^{\gamma}\right]^{1/\gamma}, \tag{3}$$

where γ parametrizes the substitutability between experience in product and process innovations, and $\psi \in [0, 1]$ the importance of product relative to process innovations for knowledge accumulation. Note that both R&D sectors benefit equally from spillovers. (We will discuss more general formulations of technology spillovers in section 6.4). If ψ is high, technical progress and growth are mainly driven by experience accumulated in product R&D. If it is low, process innovations are the main driver. The lower γ, the more complementary product and process innovations are. To save space, the main results and figures below are derived for the case $\gamma \leq 1$. Since (3) is not a production function and therefore does not need to be quasiconcave, $\gamma > 1$ is possible as well where $A(t)$ is quasiconvex in its arguments $N(t)$ and $M(t)$. We discuss the implications of the alternative parametric assumption $\gamma > 1$ after Lemma 1 and Propositions 1 and 3 below.

4 A separating equilibrium

Let us now focus on a balanced growth path in which rich households purchase all $N(t)$ goods in high quality and poor households purchase all $M(t)$ goods in low quality. We will refer to such a balanced growth path as a *separating equilibrium*. In this section we take the existence of a separating equilibrium for granted. In the next section we discuss the conditions on exogenous parameters that guarantee existence (and uniqueness) of a separating equilibrium.

The following definition characterizes a *separating equilibrium*.

Definition 1 *A* separating equilibrium *is a balanced growth path where consumption of the rich is $c_R(t) = q_h N(t)$ and consumption of the poor is $c_P(t) = q_l M(t)$. The stock of knowledge $A(t)$, the wage rate $w(t)$, the number of firms $N(t)$, and the number of mass producers $M(t)$ grow at a constant rate g. Hence also consumption of both types of consumers grows at that rate. The fraction of mass producers $m = M(t)/N(t) < 1$ and the interest rate $r(t)$ are constant over time. All $N(t) - M(t)$ exclusive producers sell only to the rich at price $p_e(t) = q_h \mu_R(t)$ and all $M(t)$ mass producers sell the low quality to the poor at price $p_l(t) = q_l \mu_P(t)$ and the high quality to the rich at price $p_h(t) = q_l \mu_P(t) + (q_h - q_l)\, \mu_R(t)$. $\mu_R(t)$ and $\mu_P(t)$ and therefore $p_e(t)$, $p_l(t)$ and $p_h(t)$ are constant over time. Both types of households have the same savings rate, so the distribution of wealth is stationary.*

4.1 Product and process innovations

In a balanced growth equilibrium, the profits of exclusive and mass producers are constant and given by $\pi_e = (1-\beta)\, L\,(q_h \mu_R - a_h)$ and $\pi_m = (1-\beta)\, L\,(q_l \mu_P + (q_h - q_l)\, \mu_R - a_h) + \beta L\,(q_l \mu_P - a_l)$. Since the interest rate r is also constant, the value of a firm that introduces a new product at date τ is given by

$$V(\tau) = \max_{\Delta} \int_{\tau}^{\tau+\Delta} \pi_e e^{-r(t-\tau)} dt + \int_{\tau+\Delta}^{\infty} \pi_m e^{-r(t-\tau)} dt - G e^{-r\Delta}.$$

To obtain the optimal timing of the process innovation Δ we use Leibniz' rule to obtain

$$\Delta = \begin{cases} 0 & \text{if} \quad (\pi_m - \pi_e)/r > G, \\ [0, \infty) & \text{if} \quad (\pi_m - \pi_e)/r = G, \\ \infty & \text{if} \quad (\pi_m - \pi_e)/r < G. \end{cases}$$

The above condition says that the present value of the increased profit flow is compared to innovation costs. We are interested in an equilibrium outcome where exclusive producers and mass producers co-exist so the first and third case of the above condition can be ruled out. This means the optimal timing of a process innovation Δ is undetermined. Hence in the simple framework discussed in the present section, firms are indifferent whether and when to invest in process innovation. However, the aggregate fraction of firms which have invested in process innovation, i.e. the fraction of mass producers m, is determined in equilibrium.[16]

[16] The indeterminacy of the individual product cycle is due to the assumption of symmetric preferences and technologies across products. This symmetry assumption provides us with a simple framework but is not critical

In the separating equilibrium, the following no-arbitrage conditions must hold

$$V_N = \frac{\pi_e}{r} = \frac{(1-\beta)L(q_h\mu_R - a_h)}{r} = F \tag{4}$$

$$V_M = \frac{\pi_m - \pi_e}{r} = \frac{L\left[q_l\mu_P - (1-\beta)q_l\mu_R - \beta a_l\right]}{r} = G.$$

Note that, along the balanced growth path, the endogenous variables μ_R, μ_P, and r are constant over time. The present value of the profit flow enabled by product innovation V_N must be equal to initial product R&D costs and the present value of the incremental profit flow enabled by subsequent process innovation V_M must be equal to process innovation costs.

A look at equations (4) reveals how the income distribution affects incentives for both product and process innovations. Larger income gaps (a lower θ) affect prices and mark-ups through its impact on μ_R and μ_P. This *price effect* raises the incentive for product innovations and reduces it for process innovations. On the other hand, higher income concentration (a larger β) changes innovation incentives also through a *market size effect*. A higher β reduces (increases) the percentage households that purchase high-quality (low-quality) goods and therefore shifts incentives towards process innovations. Hence the market size effect works against the price effect that increases rich households' willingness to pay and favors product innovations.

4.2 General equilibrium

In a balanced growth equilibrium, expenditures grow at rate g and prices are constant. Hence, consumption growth of poor and rich households follows the standard Euler equation

$$r = \sigma g + \rho. \tag{5}$$

It turns out useful to solve for the balanced growth equilibrium in terms of g, the endogenous growth rate; and p_e, the price of exclusive goods. To do so, we start from households' budget constraints. Recall that poor households are endowed with θ units of labor and $\theta v(t)$ units of firm shares and rich households are endowed with $(1-\beta\theta)/(1-\beta)$ units of labor and $[(1-\beta\theta)/(1-\beta)]\, v(t)$ units of firm shares. Hence a rich household receives an income flow $[(1-\beta\theta)/(1-\beta)]\,/\theta$ times as large as the one of a poor household.[17] The CRRA-specification of

for our results. In fact, it is straightforward to introduce asymmetries that determines the timing of process innvations Δ and generates deterministic product cycles. In section 6.2 we show that deterministic product cycles can by obtained, e.g. by allowing for learning-by-doing in production or by introducing asymmetric utilites (hierarchic preferences).

[17] The discussion here sticks to the simplifying assumption idential income compositions of rich and poor households (same θ for wage and profit income). As mentioned above, this is a special case that makes the analysis simple and transparent. The more general (and more realistic) case when income composition differs between

intertemporal preferences implies that, as long as (the distribution of) prices remain unchanged, the flow of expenditures of a rich compared to a poor household is $\left[(1-\beta\theta)/(1-\beta)\right]/\theta$ times as large, too. Because the $mN(t)$ mass producers charge price $p_h = q_l\mu_P + (q_h - q_l)\,\mu_R$ for the high quality and $p_l = q_l\mu_P$ for the low quality and because the $(1-m)\,N(t)$ exclusive producers charge price $p_e = q_h\mu_R$, the expenditure flow of a poor household is $p_l mN(t)$ and the expenditure flow of a rich household is $\left[p_h m + p_e(1-m)\right]N(t)$. The ratio of the expenditure and income flows of rich relative to a poor household is

$$\frac{mp_h + (1-m)p_e}{mp_l} = \frac{1-\beta\theta}{(1-\beta)\theta}. \tag{6}$$

Recall that $p_h = p_l + p_e\,(q_h - q_l)\,/q_h$, we can solve (6) for the relative price of exclusive to low quality products

$$\frac{p_e}{p_l} = \frac{mq_h}{q_h - mq_l}\frac{1-\theta}{(1-\beta)\theta} \tag{7}$$

This relation displays directly the price effect of inequality. Holding the share of mass products m constant, higher inequality (either lower θ or higher β) increases the relative price of exclusive products. Using the no-arbitrage conditions (4), we get an expression for the relative innovation values

$$\frac{V_N}{V_M} = \frac{(1-\beta)(p_e - a_h)}{p_l - (1-\beta)p_e q_l/q_h - \beta a_l} = \frac{F}{G}. \tag{8}$$

Equation (8) shows the price and quantity effects of higher inequality. A rising income gap through lower θ raises the middle expression in (8) since p_e increases relative to p_l, ceteris paribus. While θ has only an influence on prices, β has an additional market size effect. A rise in β, i.e. higher income concentration because of a smaller share of the rich, reduces the relative value of a product to a process innovation, given prices.

Combining (7) and (8) yields an expression for p_e as a function of m

$$p_e(m) = q_h\frac{a_l\beta/(1-\beta) - a_hG/F}{\theta\left(q_h/m - q_l\right)/\left(1-\theta\right) - q_l - q_hG/F}. \tag{9}$$

Note that $p_e(m)$ is increasing and convex over the relevant range whenever $a_l\beta/(1-\beta) > a_hG/F$.[18] The intuition for $p_e'(m) > 0$, holding inequality parameters θ and β constant, follows from the relative budget constraint and the free entry conditions: an increase in m increases

rich and poor housholds does not add economic substance to the analysis. However, in the next section, when we study transitional dynamics we need to give up this assumption since, in transition, the wealth distribution is no longer stationary.

[18]Appendix B shows that this condition must necessarily hold true in a separating equilibrium. Notice that, in equilibrium, the relative budget constraints (6) makes sure that m takes a value such that the denominator of (9) is positive.

the exclusive price p_e relative to the low-quality price p_l. Since the free entry condition of mass producers must still be satisfied, p_e must increase.

We can now represent the separating equilibrium in terms of two equations in the two unknowns g and p_e. The *first* equation derives from the no-arbitrage condition of a product innovator (4), the Euler equation (5), and the relative-expenditure equation (9)

$$g^N = \frac{L}{\sigma F}(1-\beta)\left(p_e(m) - a_h\right) - \frac{\rho}{\sigma}, \tag{10}$$

to which we refer as the *no-arbitrage curve* (N-curve). Note that the N-curve implies an upward-sloping, linear function of g in m. An increase in m, the fraction of sectors that supply low-quality mass products, is associated with a higher willingness to pay by rich households p_e. This generates a higher profit flow and raises the incentive to undertake product innovations. With a higher p_e the higher interest (and, via the Euler equation, the growth rate) has to increase, to make sure that the present value of profits settles at innovation cost F.

The *second* equation in g and m is now readily derived from the full employment condition (2). Along the balanced growth path the rich consume all $N(t)$ goods in high quality and the poor consume all $M(t)$ mass consumption goods in low quality, hence we can write $L = (1-\beta)LN(t)a_h/A(t) + \beta LM(t)a_l/A(t) + \dot{N}(t)F/A(t) + \dot{M}(t)G/A(t)$. Using the equation of motion for the aggregate stock of knowledge (3), the balanced growth conditions $g = \dot{N}(t)/N(t) = \dot{M}(t)/M(t)$ and the definition $m = M(t)/N(t)$ we get

$$g^R = \frac{L\left[\phi(m) - (1-\beta)a_h - \beta a_l m\right]}{F + Gm}, \tag{11}$$

where $\phi(m) \equiv (\psi + (1-\psi)m^\gamma)^{1/\gamma}$. Henceforth we will refer to equation (11) as the *resource curve* (R-curve). The following Lemma characterizes the slope of the R-curve.

Lemma 1 *Assume $\gamma \leq 1$. Define $D(m) \equiv \left[\beta a_l F + \phi(m)G - (1-\beta)a_h G\right] / \left[F + Gm\right]$. a) When $\phi'(1) \geq D(1)$ the slope of the R-curve is non-negative for all $m \in (0,1)$. b) When $\phi'(1) < D(1)$ there exists a unique $\hat{m}$, given by $\phi'(\hat{m}) = D(\hat{m})$, such that the slope is positive when $m < \hat{m}$ and negative when $m > \hat{m}$.*

Proof. From (11) we see that $\partial g^R/\partial m > 0$ when $\phi'(m) > D(m)$. Because $\phi'(\hat{m}) = D(\hat{m})$ we have $\partial^2 g^R(\hat{m})/\partial m^2 < 0$. Hence g^R is quasiconcave and has a unique maximum. The claims a) and b) follow. ■

Notice that the R-curve is either upward sloping or hump-shaped. On the one hand, there is a *demand effect*. An increase in m is associated with higher consumption of the poor. More employment is needed to satisfy this additional demand leaving fewer resources for research. On the other hand, there is a *productivity effect*. An increase in m means that final output is produced more efficiently, saving resources that become available for innovation and growth. The productivity effect depends on the importance of process innovation in pushing ahead the knowledge frontier, and is captured by the weight $1 - \psi$ in equation (3). The Lemma compares $\phi'(m)$, the marginal increase in m on factor productivity, to $D(m)$, the marginal effect of an increase in m on the steady-state demand for labor. The Lemma shows that the demand (productivity) effect is weak (strong) at low levels of m and becomes weaker (stronger) as m increases. The relative strength of the two effects determines whether the R-curve is positively sloped or hump-shaped. The above discussion has assumed $\gamma \leq 1$.

If $\gamma > 1$ and therefore $\phi''(m) > 0$, demand and productivity effects are still present but Lemma 1 has to be qualified in the following way: The slope of the R-curve is negative at $m = 0$ and negative $\forall m \in [0, 1)$ if $\phi'(1) \leq D(1)$. When $\phi'(1) > D(1)$, the R-curve is positively sloped near $m = 1$. Hence, with $\gamma > 1$, the resource curve might take a U-shape or even an S-form with a positive slope at values of m close to zero and one and a negative slope for intermediate values of m. As we shall see in Propositions 1 and 3 below, this does not change the qualitative predictions of inequality on growth.

4.3 The impact of inequality

We are now ready to discuss how the extent of inequality affects the prevalence of mass production and the rate of long-run growth. It turns out convenient to discuss the general-equilibrium effects of inequality in a diagram in (g, m) space. In this diagram, we draw both the N-curve and the R-curve and discuss their shifts associated with changes in the inequality parameters θ and β. Consider first the impact of larger income gaps, a smaller θ.

Proposition 1 *Consider a larger income gap, a lower θ, in a separating equilibrium with $\gamma \leq 1$. a) The percentage of mass producers m decreases. b) When the R-curve has a positive slope, $1-\psi \geq D(1)$ or $m < \hat{m}$ at the initial equilibrium, the growth rate g decreases. When the R-curve has a negative slope, $m \geq \hat{m}$ at the initial equilibrium, the growth rate g increases. c) When the R-curve has a positive (negative) slope, prices and mark-ups decrease (increase).*

The proposition says that the slope of the R-curve is crucial. When $\gamma = 1$, so the R-curve is upward sloping for small ψ and downward sloping for larger ψ.[19] When $\gamma < 1$, the case shown in Figure 4, there are two possibilities. In panel A the R-curve (11) is upward sloping for all $m \in (0, 1)$ which is the case when $1 - \psi \geq D(1)$. In panel B the R-curve is hump-shaped and refers to the case $1 - \psi < D(1)$. The N-curve (10) is unambiguously upward sloping. Note that

[19] In that case there is a critical value of $\hat{\psi} = [(1 - \beta a_l)F + (1 - \beta)a_h G] / (G + F)$ such that the R-curve is upward sloping for all m when $\psi < \hat{\psi}$ and downward sloping for all m when $\psi > \hat{\psi}$.

a reduction of θ leaves the R-curve unaffected and shifts the N-curve up. Holding m constant, a lower θ increases willingnesses to pay by the rich and the prices for exclusive goods $p_e(m)$. To make sure that the no-arbitrage condition is met, g has to increase.

Note that the effects at work are similiar if $\gamma > 1$. As discussed above, depending on parameters the R-curve is downward sloping, U-, or S-shaped. The effects of a lower θ depend again on the slope of the R-curve at its point of intersection with the N-curve. If R-curve is downward sloping, the growth rate g increases when θ decreases, and vice versa if the R-curve is upwards sloping.

Why may higher inequality induce a fall in productivity growth? Two opposing effects are at work. On the one hand, higher inequality reduces the demand for production labor associated with consumption by poor households while leaving the demand for production labor associated with consumption of rich households unaffected (the rich continue to consume all goods in high quality). On the other hand, higher inequality, by reducing m, is associated with a lower level of total factor productivity $\phi(m)$. When the former effect is dominated by the latter (in which case the R-curve slopes up), a lower θ reduces g (and increases g when the R-curve slopes down).

FIGURE 4

The following proposition explores the impact of higher income concentration, a higher β, on mass production and growth.

Proposition 2 *Consider higher income concentration – a larger β. a) When the slope of the R-curve is positive, the percentage of mass producers m decreases, while the effect on the growth rate g is ambiguous. b) When the slope of the R-curve is negative, the growth rate g and the price for exclusive goods p_e rise, while the effect on m is ambiguous.*

In Figure 5 we see that an increase in β shifts both the R-curve and the N-curve. The R-curve shifts up. This is because a higher β implies less resource-intensive consumption (there are fewer households consuming a large range of inefficiently produced high-quality goods and more households consuming a narrow range of efficiently produced low-quality goods). This releases resources available for innovation and growth. The N-curve shifts up as well. On the one hand, a higher β is associated with a smaller market for new products. On the other hand, the high concentration of income means the rich are more wealthy and willing to pay higher prices. The latter effect always dominates.

FIGURE 5

Propositions 1 and 2 suggests that the way inequality affects growth depends on the source of inequality. When growth is mainly driven by process innovations (upward-sloping R-curve) inequality reduces growth if it is due to larger income gaps, while it may increase or decrease growth if higher inequality is due to higher income concentration. What is the intuition behind the result that the two different dimensions of inequality may differentially affect the inequality-growth relationship? The reason is that higher values of both *higher* concentration (a higher β) and *smaller* gaps (a higher θ) increase the income share of the poor as a group, $\beta\theta$. The higher this share, the larger the purchasing power directed towards mass products and the higher the incentive for firms to adopt mass production technologies and open up mass markets. Notice, however, that the group income share $\beta\theta$ is not a sufficient statistic to characterize the inequality-growth relationship. The above analysis has shown that a given group income share $\beta\theta$ that arises from a large number of poor households with a very low income may have implications for growth that are quite different from the same income share that arises from as smaller number of poor households, each with a higher income.[20]

5 Existence of the separating and other equilibria

In the last section we have assumed that a separating equilibrium exists. We now study the conditions under which that equilibrium exists and will briefly discuss the situation when these conditions are violated.

5.1 Existence of a separating equilibrium

To ensure that in equilibrium a situation emerges, where a mass producer sells the high quality to the rich and the low quality to the poor, the following Lemma is helpful.

Lemma 2 *In a separating balanced growth equilibrium, the following three conditions are satisfied: (i)* $p_e (q_h - q_l)/q_h > a_h - a_l$*, (ii)* $(1-\beta)(p_e - a_h) > p_l q_h / q_l - a_h$*, and (iii)* $p_l - (1-\beta) p_e q_l / q_h - \beta a_l > 0$*.*

Proof. See Appendix A. ■

Notice that the conditions in the above Lemma are based on $p_e = q_h \mu_R$ and $p_l = q_l \mu_P$, which are endogenous. Condition (i) says that the willingness to pay of rich households for the quality

[20] Below we will briefly discuss an alternative growth regime in which the rich start to consume low-quality version as soon as mass products become available. In that case, the inequality may be harmful for growth even when growth is entirely driven by product R&D when productivity differences between exclusive and mass products are sufficiently large. In that case higher inequality implies strong detrimental average productivity effects that reduce that amout of resources available for growth. See section 5.

gap $q_h - q_l$ is sufficiently high relative to the cost gap $a_h - a_l$. In that case a mass producer strictly prefers selling the high quality to the rich and the low quality to the poor at prices p_l and $p_h = q_l \mu_P + (q_h - q_l)\mu_R$, respectively; rather than selling the low quality at price p_l to all consumers. Condition (ii) says that an exclusive firm prefers selling only to rich households at price p_e rather than selling to all households at price $p_l q_h / q_l$. Condition (iii) says that a producer with access to the mass production technology is better off separating the market (selling the low quality to the poor and the high quality to the rich) rather than selling the high quality only to the rich at a higher price p_e. Our assumption $q_l/a_l > q_h/a_h$ makes sure that (ii) and (iii) are compatible.

We are now ready to establish the existence of a separating equilibrium. It turns out easier to work with p_e instead of m as endogenous variable. Define the function $m(p_e)$ as the inverse of $p_e(m)$ from (9).

Proposition 3 *A separating balanced growth equilibrium determined by the intersection of the two curves (10) and (11) exists if (i) $g^N(m(p_e^1)) < g^R(m(p_e^1))$, (ii) $g^N(m(p_e^2)) > g^R(m(p_e^2))$, and (iii) $p_e^2 > p_e^1$ where $p_e^1 \equiv (a_h - a_l)q_h/(q_h - q_l)$ and $p_e^2 \equiv a_h + \beta/(1-\beta) \cdot (F/G) \cdot (a_h/q_h - a_l/q_l)q_l$.*

Proof. See Appendix B. ■

To prove Proposition 3 we need to check at which values of p_e and p_l the inequality conditions of Lemma 2 hold. We exploit the fact that p_l is a linear function of p_e since equation (7) must hold. The proof of Proposition 3 shows that the point of intersection of the two curves has to be at a value of p_e between p_e^1 and p_e^2. This is the case for the N-curve (10) and the R-curve (11),[21] when the N-curve lies below the R-curve at $m(p_e^1)$ and is above the R-curve at $m(p_e^2)$. In this case the equilibrium exists and is unique when $\gamma \leq 1$. When the R-curve lies below the N-curve both at p_e^1 and p_e^2, there might be still a separating equilibrium but it is no longer unique even for $\gamma < 1$. The Proposition also holds for the case $\gamma > 1$. However, with $\gamma > 1$, the R-curve must be downward sloping to guarantee uniqueness.

5.2 Alternative balanced growth equilibria

Along a balanced growth equilibrium, one of the following four outcomes may emerge. Case (1): the separating equilibrium studied above. Case (2): exclusive firms sell only to the rich and mass producers sell the low quality to all households. Case (3): mass producers do not exist;

[21]Note that, with $\gamma \leq 1$, the resource curve is concave when upward sloping. Instead, with $\gamma > 1$, an upward sloping resource-curve might be convex, giving rise to potential multiple equilibria, see discussion in the main text below. The no-arbitrage curve is convex, since $p_e(m)$ is a convex function of m.

active firms supply only the high quality. Case (4): exclusive firms do not exist; active firms supply only the low quality. Here we only briefly discuss cases (2) - (4).

The separating regime arises under parameter constellations ensuring that the conditions of Lemma 1 hold. Above we have seen that this regime is more likely (i.e. the gap between p_e^1 and p_e^2 is sufficiently large), if (i) the quality gap $q_h - q_l$ is high (but not too high) relative to the cost gap $a_h - a_l$ and (ii) if process innovation costs G are low. In constrast, when process innovation costs are high and/or the quality gap very high or very low, alternative equilibria arise. Figure 6 provides a graphical illustration for these alternative growth equilibria.

FIGURE 6

In this subsection we briefly describe each of these alternative balanced growth regimes.

Rich households purchase low-quality mass producers. This alternative equilibrium emerges when the quality differential is small. In the extreme case, when a process innovation only changes cost of production but leaves the quality of an existing product unchanged, mass producers are not able to separate the market. Instead they will sell the same product to both rich and poor households charging the price that the poor can afford. In this alternative equililibrium, the first inequality of Lemma 1 is reversed. Instead we have $p_e(q_h - q_l)/q_h < a_h - a_l$. In that case, a mass producer is better off supplying only the low quality at price p_l and not supplying the high quality at all. Exclusive firms continue to sell the high-quality product to the rich at price p_e. Conditions (ii) and (iii) of Lemma 1 remain valid.

Along the balanced growth path of such an equilibrium the N-curve and the R-curve are given by

$$g^N = \frac{L(1-\beta)\left[\tilde{p}_e(m) - a_h\right]}{\sigma F} - \frac{\rho}{\sigma}, \text{ and}$$
$$g^R = \frac{L\left[\phi(m) - (1-m)(1-\beta)a_h - ma_l\right]}{F + Gm}.$$

where $\tilde{p}_e(m)$ is the price of exclusive goods that is associated with no-entry conditions and household budget constraints in the present equilibrium where mass goods are purchased in low quality also by the rich.[22]

Just like in a separating equilibrium, the inequality-growth relationship depends crucially on the shape of the R-curve. In particular, the slope of the R-curve is either upwards sloping

[22] The relationship between the price of exclusive goods and the percentage mass producers differs from the one of the separating equilibrium. Consumption expenditures by rich consumers are now $(1-m)p_e + mp_l$ and profits of mass producers are now $\pi_m = (p_l - a_l)L$. To derive the balanced-growth relation between the price of exclusive goods and the fraction of mass producers we proceed in a similar way as in the separating equilibrium. We get $\tilde{p}_e(m) = \left[a_l - (1 + G/F)(1-\beta)a_h\right]\left[(1/m - 1)(1-\beta)\theta/(1-\theta) - (1-\beta)(1+G/F)\right]^{-1}$.

or hump-shaped, depending on the relative strength of demand and knowledge spillover effects of mass production. Denote by $\tilde{D}(m)$ the increase in labor demand associated with a marginal increase in the number of mass producers. It is straightforward to see that the R-curve is upward sloping, if $1-\psi > \tilde{D}(1) = -[(1-\beta)a_h - a_l] + (1-a_l)G/(F+G)$. The following proposition describes the relationship between inequality, the direction of change, and long-run growth in this alternative equilibrium.

Proposition 4 *a) When $\tilde{D}(1) < 0$, higher income concentration (a larger β) and/or smaller income gaps (a larger θ) increase growth g and mass production m, irrespective of the importance of process-innovations for knowledge spillovers $1-\psi$. b) When $\tilde{D}(1) \geq 0$ the inequality-growth relationship is qualitatively similar to the one of the separating equilibrium.*

Part a) of the above proposition states that, in the alternative mass production equilibrium, there are parameter constellations such that the demand effect dominates the knowledge-spillover effect, even when all knowledge spillovers arise from product innovations (and process innovations do not drive productivity). This is the case when $(1-\beta)a_h - a_l > (1-a_l)G/(F+G)$ so that an increase in the percentage mass producers is *resource-saving,* i.e. the marginal effect on the demand for labor of an increase in the balanced-growth value of m saves resources to the economy. In other words, $\tilde{D}(m) < 0$ for all m. The R-curve is upwards sloping, whatever the value of the knowledge-spillover parameter ψ, even when knowledge spillovers arise only from product innovations. Intuitively, this case arises with a large gap in productivity-levels between exclusive and mass production (a_l and G are small, a_h and F are large) and when the population share of the rich $1-\beta$ is large (so that exclusive firms operate on a large scale). Under these conditions, shifting demand away from exclusive towards mass production releases resources for innovation and growth. As a result, changes in parameters that lead to an extension of mass production are always associated with higher growth.

Part b) of the proposition states that, when $\tilde{D}(1) > 0$, the alternative mass production equilibrium behaves very similar to the separating equilibrium. In particular, the R-curve is either upward sloping or hump-shaped, depending on the relative strength of demand and knowledge-spillover effect. When $1-\psi \leq \tilde{D}(1)$ the R-curve is upward-sloping between $(0, \tilde{m})$ and downward sloping between $(\tilde{m}, 1)$. When the initial equilibrium is at $m \leq \tilde{m}$ a higher β and/or a higher θ increase both g and m. When the initial equilibrium is at $m > \tilde{m}$ a higher θ increases m and decreases g; a higher β also increase m and has an ambiguous effect on g.

Degenerate equilibria. With other paramater values, two other types of equilibria may emerge. When G is prohibitively high, mass production technologies will not be adopted at all.

When G is close to zero, mass production technologies will be adopted immediately by all firms. In these degenerate equilibria, the distinction between mass and exclusive produers vanishes and the model becomes equivalent to the case of expanding product varieties. As shown in Foellmi and Zweimüller (2006), in such equilibria higher inequality (lower θ and/or higher β) unambiguously increases growth. The balanced growth path is such that some firms sell their product only to the rich at a high price and other firms sell to the whole customer base at a low price; in equilibrium the two options yield the same profit. Higher inequality (lower θ and/or higher β) raises growth because it always releases resources: it reduces consumption of the poor while leaving consumption of the rich unchanged (they still consume all goods).

6 Extensions

The above analysis has made a number of stylized assumptions. In particular, we have assumed that the high- and the low-quality products are produced by the same firm; that goods are symmetric both with respect to technology and with respect to preferences; and that there are only two groups of consumers. We now show that our model remains still tractable and yields additional insights when we relax these assumptions.

6.1 Market structure and competition

We first relax the assumption that both high- and low-quality version of a given product line are produced by the same firm. While this assumption simplifies the analysis it abstracts from important phenomena in reality. In many cases, including the automobile, new products and subsequent mass products were produced by different firms competing with each other.

Let us now assume that not only the incumbent but also outsiders may pay the fixed cost G and develop the Model T version of an existing product. It is easy to see that the outsider has a higher incentive than the incumbent to undertake process R&D. The reason is straightforward. Entry with the low-quality version of an existing product yields a profit flow $\pi_o = \beta L(p_l - a_l)$ to the outsider. In contrast, introducing a low-quality version yields an incremental profit flow for the incumbent given by $\pi_m - \pi_e = \beta L(p_l - a_l) - (1 - \beta)(p_e - p_h) < \pi_o$. The reason is that introducting a low-quality version forces the incumbent to reduce prices for the high-quality version from p_e to $p_h < p_e$ to make sure that the rich are still willing to purchase the high quality. For an entrant, such a “cannibalization” effect does not occur.

Introducing the possibility of entry and opening up mass markets by an outsider implies that there will be two suppliers of a product line in equilibrium: the intial innovator who continues to supply the high-quality and the new entrant offering the low-quality version. The pricing problem is then an infinitely repeated game between the incumbent and the entrant. Here we restrict ourselves to a subgame perfect Nash equilibrium which maximizes the profits of both firms in the stage game. (This could be supported, e.g., through trigger strategies). In this simple case the prices for the high- and low-quality product take the same form as in the basic (monopoly) model. In equilibrium, there are $mN(t)$ product lines with a duopolistic market structure and $(1-m)N(t)$ product lines served by monopolists, exclusively in high quality.

The respective values of a product innovation and a process innovation become

$$
\begin{aligned}
V_N &= \frac{(1-\beta)L\left[(p_e - a_h) - m^{r/g}\left(p_e q_l/q_h - p_l\right)\right]}{r} = F, \text{ and} \\
V_M &= \frac{\beta L\left[p_l - a_l\right]}{r} = G.
\end{aligned}
$$

Notice that the value of a product innovation V_N in an oligopolistic economy is lower than the value of a product innovation in the monopoly case. This can be seen directly from comparing V_N in the monopoly case, see equation (4), to the above expression for V_N. This implies that the N-curve shifts downwards in (g, m)-space. Notice further that allowing for entry-competition leaves the R-curve (11) unchanged.

This reveals an interesting prediction on the relationship between market structure and growth. Comparing the oligopolistic to the monopolistic economy, the percentage of mass production sectors is higher in the oligopolistic economy, while growth can be higher or lower. When the initial equilibrium is on the upward sloping branch of the R-curve, a oligopolistic economy has a higher balanced growth rate than an otherwise identical monopolistic economy, and vice versa when R-curve is downward sloping. This implies a hump-shaped relationship between competition and growth in the following sense. When there is potential competition in few sectors (m is low) growth is larger in an oligopolistic compared to a monopolistic economy; when there is potential competition in many sectors (m is high) growth is lower in an economy with entry-competition.

6.2 Product cycles

In the above model we have assumed that all product lines are ex-ante symmetric. While the framework determines endogenously the percentage mass producers, the product cycle of an individual good remains indeterminate. (Recall from section 4.1 above that the timing of process innovations remains undetermined.) With deterministic product cycles, however, the "life" of a product is split up into a period of exclusive production with deterministic length followed by an infinitely long period of mass production. We discuss two ways that account for deterministic cycles. The first extension introduces asymmetries on the cost side through learning from production experience. A second extension discusses asymmetries on the demand side due to hierarchical preferences.

6.2.1 Learning-by-doing

Suppose that production experience leads to falling labor requirements in production, so that there is learning-by-doing at the level of the individual firm. Assume that $a_k(j,t)$, the labor input necessary to produce good j at date t in quality $k \in \{h,l\}$ is determined by

$$a_k(j,t) = (1-\Lambda(j,t))a_k/A(t), \quad \Lambda(j,t) = \int_{-\infty}^{t} \delta x(j,s)\exp(-\delta(t-s))ds,$$

where δ is the speed of learning as well as the depreciation rate of learning capital; and $x(j,t) \in \{0, 1-\beta, 1\}$ is the quantity of production by firm j at date t.[23] The labor productivity of a firm now increases for two reasons: a general spillover due to past product and process innovations; and through individual manufacturing experience. Our learning formulation assumes that high- and low-quality goods production contributes in the same way to increases in productivity.

The optimal "age" of a product at which the firm implements the process innovation and open up the mass market is given by

$$\begin{aligned}
\max_{\Delta} V(j,t) &= \int_{t}^{t+\Delta} \pi_e(j,s)\exp(-r(s-t))ds + \int_{t+\Delta}^{\infty} \pi_m(j,s)\exp(-r(s-t))ds - G\exp(-r\Delta) - F, \\
\pi_e(j,s) &= (1-\beta)L\left[p_e - (1-\Lambda(j,t))a_h\right], \\
\pi_m(j,s) &= (1-\beta)L\left[p_h - (1-\Lambda(j,t))a_h\right] + \beta L\left[p_l - (1-\Lambda(j,t))a_l\right].
\end{aligned}$$

To see that the product cycle now becomes deterministic, we compare the optimal date of switching from exclusive to mass production. From the point of view of an exclusive producer, the difference between (hypothetical) profit flow from mass production and the (actual) profit flow from exclusive production, $\pi_m(j,s) - \pi_e(j,s)$, is increasing over time. This is because, initially, production costs are high and can be covered only by high prices that the rich are willing to pay but, over time, costs decrease making mass production increasingly attractive. The optimal product age of implementing mass production, Δ, is reached when $\pi_m(j,\Delta) - \pi_e(j,\Delta) = rG$. When we consider the cross-section of firms at a point in time, it is the "oldest" firm that has the lowest cost of production. It is then always the oldest exclusive firm, that has the lowest production costs and which has the highest incentive to implement mass production.

The equilibrium is solved in an analoguous way as in the basic model. We use the zero-profit conditions, combine them with the Euler equation (5) and the household budget constraints to derive the N-curve in m and g. With learning-by-doing, the R-curve is now given by

$$g^R = \frac{L\left[\phi(m) - (1-\beta)A_h(m)(1-m) - A_l(m)m\right]}{F + Gm}$$

[23]Our formulation of learning-by-doing borrows from Matsuyama (2002).

$A_l(m) = ((1-\beta)a_h + a_l)\int_0^{mN(t)}(1-\Lambda(j,t))dj/A(t)$ and $A_h(m) = a_h \int_{mN(t)}^{N(t)}(1-\Lambda(j,t))dj/A(t)$ are average learning coefficients (that discount labor requirements a_h and a_l) of mass producers and exclusive producers, respectively. It can be shown that both $A_l(m)$ and $A_h(m)$ are constant along the balanced growth path.

While the basic mechanisms that drive the inequality-growth relationship remain unchanged, some interesting qualitative differences emerge. Most importantly, learning-by-doing implies that the R-curve is more likely positively sloped. This is because learning is highest in mass production sectors where the scale of production is large. Computations show that, depending on the strength of learning-by-doing δ, the R-curve may be rising or falling in m even when product innovations are the main driver of growth. Similar to the basic model, larger income gaps (a fall in θ) raise prices and decreases mass markets m, which tends to increase resources available for R&D (provided productivity-spillovers from process innovations are not too large). However, larger income gaps are associated with more exclusive production and hence less firm-specific learning. As a result, even when the innovations spillovers are entirely driven by product innovations, larger income gaps become harmful for growth when learning-effects are strong (δ is large). For a similar reason, higher income concentration is more favorable for growth than in the basic model provided it is associated with higher mass production.

6.2.2 Hierarchic preferences

Learning by doing provides one example by which deterministic product cycles can be generated. Another way to obtain deterministic cycles is by introducing asymmetric utilities. More precisely, assume a *hierarchy of consumption* where *basic* goods yield high utility (i.e. they have high priority in consumption because they cover more basic needs) and more luxurious goods yield lower utilities (i.e. are consumed only when the more basic needs have already been satisfied).

The utility function takes the following form

$$u(t) = \int_0^{N(t)} \xi(j)x(j,t)q(j,t)dj,$$

where we have added a *hierarchy weight* $\xi(j)$ which is strictly monotonically decreasing in j. Hence low-j goods get a higher weight than high-j goods, and thus households have a higher willingness to pay for low-j than for high-j goods. Product innovation R&D will focus on the lowest-j good that has not yet been invented. Process R&D will be targeted towards the exclusive good that has highest priority (the lowest-j among the exclusive goods). For balanced growth, hierarchy weights need to be a power function, $\xi(j) = j^{-\eta}$ (see Bertola, Foellmi, and Zweimüller, 2006, Chapter 12).

The optimal timing of the process innovation Δ is determined by the solution the problem

$$\begin{aligned}
\max_{\Delta} V(j,t) &= \int_t^{t+\Delta} \pi_e(j,s)\exp(-r(s-t))ds + \int_{t+\Delta}^{\infty} \pi_m(j,s)\exp(-r(s-t))ds - G\exp(-r\Delta), \\
\pi_e(j,s) &= L(1-\beta)\left[j^{-\eta}p_e(s) - a_h\right], \\
\pi_m(j,s) &= L\left[\beta\left(j^{-\eta}p_l(s) - a_l\right) + (1-\beta)\left(j^{-\eta}\left(\frac{q_h - q_l}{q_h}p_e(s) + p_l(s)\right) - a_h\right)\right].
\end{aligned}$$

The profit flows of some good j depends on the relative position, $j/N(t)$, in the consumption hierarchy. For good j, the relative position in the hierachy decreases (i.e. the goods gets more and more priority) hence prices increase over time. It can be shown that $p_e(t)$ and $p_l(t)$ are increasing at rate ηg.[24] Initially the difference between the profit flow from mass production is small relative to the one from mass production.[25] However, the difference narrows and the optimal date Δ of undertaking process R&D and switching to mass production has come when $\pi_m(j,\Delta) - \pi_e(j,\Delta) = rG$. Note that if we let $\eta \to 0$, the hierarchic preferences formulation converges to the basic (symmetric) model. Instead, if η is sufficiently high, the "innovate-and-wait" pattern arises (studied in Foellmi and Zweimüller, 2006). In this regime, firms innovate early to secure a patent, and then wait for a certain period before the rich are willing to purchase this new product.

6.3 A middle class

The above analysis assumes only two groups of consumers, rich and poor. What happens if there is a middle class? This question is potentially interesting because it is often argued that a large and wealthy middle class is a prerequirement for the emergence of mass consumption societies. In a separating equilibrium, a single consumer will never consume two different qualities in steady state. This argument implies that, when there are three classes, two types of equilibria may emerge. In the *first type of equilibrium*, middle class households are relatively wealthy. They consume only high-quality goods, and purchase some (though not all) goods supplied by exclusive producers; and all goods by mass producers. In this case, middle class households are like the rich, albeit a bit less wealthy.

How will a redistribution from the poor to the middle class affect mass production and growth? When income is redistributed from the *poor to the middle class*, the effects are similar

[24] In order that the no-arbitrage condition holds, the initial present value of every newly set up firm must equal F. Hence the hierarchy-independent part of the willingness-to-pay $\mu_i(t)$ must rise at $-\partial/\partial t\left(j^{-\eta}\right) = \eta g$ over time, in order that the overall willingness to pay for a good only depends on the time span since inception, and not on time.

[25] The revenues of mass producers must be higher in equilibrium. Otherwise, firms would never switch to mass strategies given process innovation costs. Note also that both revenue streams grow at the same rate. It follows that $\pi_m(j,s) - \pi_e(j,s)$ grows over time.

to a redistribution of income from poor to rich in the separating equilibrium. Demand is shifted towards exclusive producers raising the incentive to undertake product R&D. However, unlike in the two-class society, such a redistribution also increases the demand for production labor, because the middle class increases exclusive consumption. Hence the effect on growth of a redistribution from the poor to the middle class is more harmful than a redistribution from bottom to top in the two class-society. When income is redistributed from the *rich to the middle class,* the incentive for product innovation falls because the rich reduce their willingness to pay. Moreover, such a redistribution increases the demand for labor in production (because the rich continue to consume all goods but the middle class consumes more). Hence such a redistribution reduces growth when product innovations are the main driver of growth. When process innovations are the driving force, the effect on growth is ambiguous (N-curve and R-curve shift in opposite directions).

In the *second type of equilibrium*, middle class households are less wealthy and cannot afford products supplied by exclusive producers. Instead, they consume all products supplied by mass producers and consume all these goods in low quality. The poor can afford only a subset of the low-quality mass products. A redistribution from the *rich to the middle class* shifts innovation incentives towards process R&D generating very similar effects to those that are present in a two-class society. However, there is an additional effect. The redistribution increases the willingness to pay of the middle class driving up prices for low-quality goods. This induces poor households to consume less which saves resources for R&D. Due to this resource-saving effect, such a redistribution is more likely to be beneficial for growth than an income transfer from rich to poor in the two-class society. Finally, a redistribution from the *poor to the middle class* raises the willingness to pay for low-quality products and drives up prices for low-income goods. The poor consume less both because of the direct effect of lower income and because of the indirect effect of higher prices. In sum, innovation incentives shift towards process innovations and resources are saved.

6.4 Biased knowledge spillovers

Our assumptions on *technology and technical progress* so far have assumed that knowledge spillovers from process and product innovations increase productivity, both in the R&D sectors and in production. We have assumed that the productivity ratios are exogenous as the labor inputs both in production and R&D are inversely proportional to the aggregate knowledge stock, $A(t)$. When knowledge stocks in exclusive and mass production are determined in different ways (i.e. product- and process-innovations have different impacts on TFP in exclusive and mass production), the analysis gets more complicated because productivity gaps become endogenous. To avoid this complication, let us assume that knowledge spillovers take place only within (mass and exclusive) sectors, so that $\tilde{F}(t) = F/N(t)$, $\tilde{a}_h(t) = a_h/N(t)$, $\tilde{G}(t) = G/M(t)$, $\tilde{a}_l(t) = a_l/M(t)$. Then the growth rate is readily determined by resource constraint (11)

$$g = \frac{L\left[1 - (1-\beta)a_h - \beta a_l\right]}{F + G}.$$

It turns out that a higher concentration of income (higher β) has a positive impact on growth but changes in income gaps as measured by θ do not have any impact on growth. Note, however, that changes in θ affect the percentage of mass producers m.[26] However, as soon as there are knowledge spillovers from product R&D to process R&D and productivity in mass production and/or from process R&D to product R&D and productivity in exclusive production, the inequality-growth relationship remains qualitatively similar to the one in the basic model.

6.5 Divisible products

Similarly, our assumptions on *preferences* was very stylized and emphasizes exclusively the extensive margin of consumption. Is the indivisibility assumption crucial? The answer is no. Assume that goods are perfectly divisible so that consumers face both the choice whether or not to consume a product and, if yes, in which quantity. Let us assume a general felicity function that is additively separable in the various products

$$u(t) = \int_0^{N(t)} v(c(j,t))dj$$

To generate a situation where consumers react both along the extensive and the intensive margin the subutility $v(.)$ has to be such that the marginal utility from consuming the first unit is finite, $v'(0) < \infty$. This generates a finite reservation price that is higher for rich consumers. With a general subutility $v(.)$, the implied demand curves for the individual household will feature changing price elasticities of demand. Unless the subutility $v(.)$ belongs to the HARA class, the income distribution will affect demanded quantities even when a product is purchased by all households. In that case, Engel-curves are no longer linear, and market demand curves depend on the distribution of income even in a symmetric equilibrium.[27]

[26] Globally, the extent of inequality will not be completely irrelevant, because inequality still determines the growth regime, i.e. whether we are in a separating balanced growth equilibrium or some other balanced growth equilibrium (see section 5 above).

[27] Foellmi and Zweimüller (2004) analyze the impact of inequality on mark-ups in the context of a symmetric full employment equilibrium. It turns out that it depends on the curvature of the coefficient of absolute risk aversion, $-v''(c)/v'(c)$, whether higher inequality in the size distribution of income increases or decreases the mark-up.

7 Conclusions

In this paper we presented an endogenous growth model where firms invest both in product and process innovations. Product innovations (that open up completely new product lines) satisfy the luxurious wants of the rich. Process innovations (that decrease costs per unit of quality) transform the luxurious products of the rich into conveniences of the poor. A prototypical example for such a situation is the automobile. Initially an exclusive product for the very rich, the automobile became affordable to the poorer classes after the introduction of Ford's Model T, the car that "put America on wheels".

Our analysis explores how different dimensions of inequality affect the incentives for product and process innovations when consumers have non-homothetic preferences. It turns out that alternative dimensions of inequality (income gaps versus income concentration) affect the demand for new luxuries and mass products in quite different ways. The introduction of non-homotheticicities implies that the distinction between product and process innovations becomes crucial. In standard R&D based growth models the distinction between product and process innovations is of minor relevance. This is different in our framework. On the one hand, product and process innovations are affected differentially by changes in the distribution of income and wealth across households; on the other hand, a change in the composition of innovative activities affects rich and poor consumers in different ways. As a consequence, the welfare effects of growth policies depend in a crucial way on the particular source that drives economic growth.

The mix of innovative activities is also the crucial issue in models of directed technical change. In contrast to this literature, which focuses on the relative demand and relative productivities of production factors, our model focuses on the relative demand and relative prices for alternative types of consumer goods (exclusive goods versus mass products). Hence our model highlights inequality across households as a potentially important determinant of the composition of R&D, a channel that is absent in directed technical change models.

Our framework is simple and tractable, and lends itself nicely to studying a broader set of issues relevant for the inequality-growth relationship. We sketch how competition affects growth (when the mass product is introduced by an outsider who competes with an incumbent exclusive producer); and the role of learning-by-doing and hierarchic preferences (that introduces heterogeneity across firms and accounts for product cycles). Presumably, our model is also potentially useful to explore further issues, such as the welfare consequences of globalization and international trade (both within and between countries), the role of quality upgrading (such that subsequent mass products can be of much higher quality than exclusive products at the date of invention) or the implications of skill-biased technical change in the presence of non-homothetic preferences (through their impact on the supply of and the demand for new products and processes). We think these are interesting directions for future research.

References

[1] Acemoglu, Daron (1998), "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality," *Quarterly Journal of Economics* 113, 1055-1090.

[2] Acemoglu, Daron (2002), "Directed Technical Change," *Review of Economic Studies* 69, 781-809.

[3] Acemoglu, Daron (2009), "*Introduction to Modern Economic Growth,*" Princeton and Oxford: Princeton University Press.

[4] Acemoglu, Daron and Fabrizio Zilibotti (2001), "Productivity Differences," *Quarterly Journal of Economics* 116, 563-606.

[5] Aghion, Philippe and Peter Howitt (1992), "A Model of Growth Through Creative Destruction," *Econometrica* 60, 323-351.

[6] Bertola, Giuseppe, Foellmi Reto, and Josef Zweimüller (2006), "*Income Distribution in Macroeconomic Models,*" Princeton and Oxford: Princeton University Press.

[7] Bowden, Sue and Avner Offer (1994), "Household Appliances and the Use of Time: The United States and Britain Since the 1920s," *The Economic History Review* 47, 725-748.

[8] Brunner, M. and H. Strulik (2002), "Solution of Perfect Foresight Saddlepoint Problems: A Simple Method and Applications," *Journal of Economic Dynamics and Control* 26, 737-753.

[9] Falkinger, Josef (1994), "An Engelian Model of Growth and Innovation with Hierarchic Consumer Demand and Unequal Incomes," *Ricerche Economiche* 48, 123-139.

[10] Falkinger, Josef and Josef Zweimüller (1996), "The Cross-country Engel Curve for Product Diversification," *Structural Change and Economic Dynamics* 7, 79-97.

[11] Foellmi, Reto and Josef Zweimüller (2004), "Inequality, Market Power, and Product Diversity," *Economics Letters* 82, 139-145.

[12] Foellmi, Reto and Josef Zweimüller (2006), "Income Distribution and Demand-Induced Innovations," *Review of Economic Studies* 73, 941-960.

[13] Grossman, Gene and Elhanan Helpman (1991), "*Innovation and Growth in the Global Economy,*" Cambridge, MA: MIT Press.

[14] Hounshell, David A. (1984), “*From the American System to Mass Production, 1800-1932, The Development of Manufacturing Technology in the United States*,” Baltimore and London: John Hopkins University Press.

[15] Li, Chol-Won (2003), ”Income Inequality, Product Market, and Schumpeterian Growth” (Mimeo, University of Glasgow).

[16] Jackson, Laurence F. (1984), ”Hierarchic Demand and the Engel Curve for Variety,” *Review of Economics and Statistics* 66, 8-15.

[17] Katona, George, ”*The Mass Consumption Society*,” New York: McGraw-Hill, 1964.

[18] Krugman, Paul (1979), ”A Model of Innovation, Technology Transfer, and the World Distribution of Income,” *Journal of Political Economy* 87, 253-66.

[19] Nagaoka, S. and J. P. Walsh (2009), ”The R&D Process in the US and Japan: Major Findings from the RIETI-Georgia Tech Inventor Survey”, *RIETI Discussion Paper* Series 09-E-010.

[20] Matsuyama, Kiminori (1999), ”Growing Through Cylces,” *Econometrica* 67, 335-347.

[21] Matsuyama, Kiminori (2000), ”A Ricardian Model with a Continuum of Goods under Nonhomothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade,” *Journal of Political Economy* 108, 1093-1120.

[22] Matsuyama, Kiminori (2002), ”The Rise of Mass Consumption Societies,” *Journal of Political Economy* 110, 1035-1070.

[23] Mokyr, Joel (1999), ”The New Economic History and the Industrial Revolution,” in Mokyr, ed., *The British Industrial Revolution: an Economic Perspective*, Boulder: Westview Press, 2nd ed., 1-127.

[24] Moriguchi, Chiaki and Emmanuel Saez (2006), ”The Evolution of Income Concentration in Japan, 1886-2002: Evidence from Income Tax Statistics,” *NBER Working Papers* 12558.

[25] Murphy, Kevin M., Andrei Shleifer, and Robert Vishny (1989), ”Income Distribution, Market Size, and Industrialization,” *Quarterly Journal of Economics* 104, 537-564.

[26] Perkin, Harold (1969), “*The Origins of Modern English Society, 1780-1880*,” London: Routledge & Kegan Paul Ltd.

[27] Romer, Paul M. (1990), ”Endogenous Technological Change,” *Journal of Political Economy* 98 (IIC), 71-102.

[28] Schmookler, Jacob (1966), "*Invention and Economic Growth*," Cambridge, MA: Harvard University Press.

[29] Zweimüller J. and J. K. Brunner (2005), "Innovation and Growth with Rich and Poor Consumers," *Metroeconomica* 56, 233-262.

Appendix A: Proof of Lemma 2

A mass producer selling the high quality to the rich and the low quality to the poor faces the following profit maximization problem:

$$\max_{p_h,p_l} \left[L(1-\beta)(p_h - a_h) + L\beta(p_l - a_l)\right],$$

s.t. (i) $p_h \leq q_h\mu_R$, (ii) $p_l \leq q_l\mu_P$, (iii) $q_h\mu_R - p_h \geq q_l\mu_R - p_l$, and (iv) $q_l\mu_P - p_l \geq q_h\mu_P - p_h$,

The constraints are based on the first-order conditions of households (1). (i) and (ii) ensure that households purchase the good (rationality constraints), and (iii) and (iv) ensure that rich households prefer to buy the high quality and poor the low (incentive constraints). Notice that a firm cannot separate the rich into the low quality and the poor into the high given the higher willingness to pay of the rich, $\mu_R > \mu_P$.[28]

Constraint (iii) and $\mu_R > \mu_P$ imply $q_h\mu_R - p_h \geq q_l\mu_R - p_l > q_l\mu_P - p_l$. Hence if constraint (ii) were inactive, so would be (i). But then the firm could increase both prices by the same amount without violating (iii) and (iv). Hence constraint (ii) must be active, $q_h\mu_R - p_h \geq q_l\mu_R - p_l > q_l\mu_P - p_l = 0$, which implies that constraint (iii) must be active, too. Otherwise the firm could increase the price of the high quality without violating constraints (iii) and (i). Since constraint (iii) is active, $q_h\mu_R - p_h = q_l\mu_R - p_l > q_l\mu_P - p_l = 0$, constraint (i) cannot be active. Rewritting the active constraint (iii), $p_h - p_l = q_h\mu_R - q_l\mu_R > q_h\mu_P - q_l\mu_P$ shows that constraint (iv) is not active as well. Hence constaints (ii) and (iii) are active, $p_l = q_l\mu_P$ and $q_h\mu_R - p_h = q_l\mu_R - p_l$, and a separating mass producer optimally sets prices $p_l = q_l\mu_P$ and $p_h = q_l\mu_P + (q_h - q_l)\mu_R$.

Recall that a mass producer has four other options besides separating the rich into the high quality and the poor into the low (h,l): sell the high quality only to rich $(h,0)$ or to all households (h,h), or sell the low quality only to rich $(l,0)$ or to all households (l,l). The five options yield the following profit flows:

$$\begin{aligned}
\pi_{h,0} &= L(1-\beta)(q_h\mu_R - a_h), \\
\pi_{h,h} &= L(q_h\mu_P - a_h), \\
\pi_{h,l} &= L\beta(q_l\mu_P - a_l) + L(1-\beta)((q_h - q_l)\mu_R + q_l\mu_P - a_h), \qquad (12) \\
\pi_{l,l} &= L(q_l\mu_P - a_l), \\
\pi_{l,0} &= L(1-\beta)(q_l\mu_R - a_l).
\end{aligned}$$

It is easy to verify that if the conditions in Lemma 2 hold, separating households (h,l) is an optimal strategy for mass producers. Condition (i) $(q_h - q_l)\,\mu_R > a_h - a_l$ ensures that selling the

[28] Incentive constraints of $q_l\mu_R - p_l \geq q_h\mu_R - p_h$ and $q_h\mu_P - p_h \geq q_l\mu_P - p_l$ would require $(q_h - q_l)\mu_P \geq p_h - p_l \geq (q_h - q_l)\mu_R$, which cannot hold.

low quality to all households (l, l) yields lower profits. Condition (iii) $q_l \mu_P - (1-\beta) q_l \mu_R - \beta a_l > 0$ ensures that selling only the high quality to rich households $(h, 0)$ yields lower profits. And since condition (ii) $(1-\beta)(q_h \mu_R - a_h) > (q_h \mu_P - a_h)$ ensures that exclusive producers prefer selling the high quality only to rich households instead to all, selling the high quality to all households (h, h) must generate lower profits for mass producers, as well. And finally, condition (i) also ensures that selling the low quality only to rich households $(l, 0)$ is inferior (to selling the high quality only to rich households and thus to separating households). Similarly for exclusive producers which can only supply the high quality, condition (ii) ensures that selling only to rich households is an optimal strategy.

If conditions (ii) and (iii) in Lemma 2 hold with strict inequality, exclusive producers sell only to the rich generating $\pi_e = \pi_{h,0}$, and mass producers separate households generating $\pi_m = \pi_{h,l}$. Would condition (ii) holds with equality instead, exclusive firms were indifferent between selling only to rich and to all households, $\pi_e = \pi_{h,0} = \pi_{h,h}$. Would condition (iii) hold with equality instead, mass producers were indifferent between selling only to rich and selling to all, separating households, $\pi_m = \pi_e$.

Appendix B: Proof of Proposition 3

We show that a balanced growth equilibrium determined by (10) and (11) exists if the conditions in Lemma 2 hold. Using equation (7) to express μ_P as a linear function of $\mu_R = p_e/q_h$, it is easy to see that the conditions in Lemma 2 only hold if $p_e \in (p_e^1, p_e^2)$. A balanced growth equilibrium exists and is unique if (10) and (11) cross for a value of $p_e \in (p_e^1, p_e^2)$. Because of the different slopes of the two equilibrium curves, this holds true as long as the N-curve lies below (above) the R-curve at p_e^1 (p_e^2). Therefore we must have

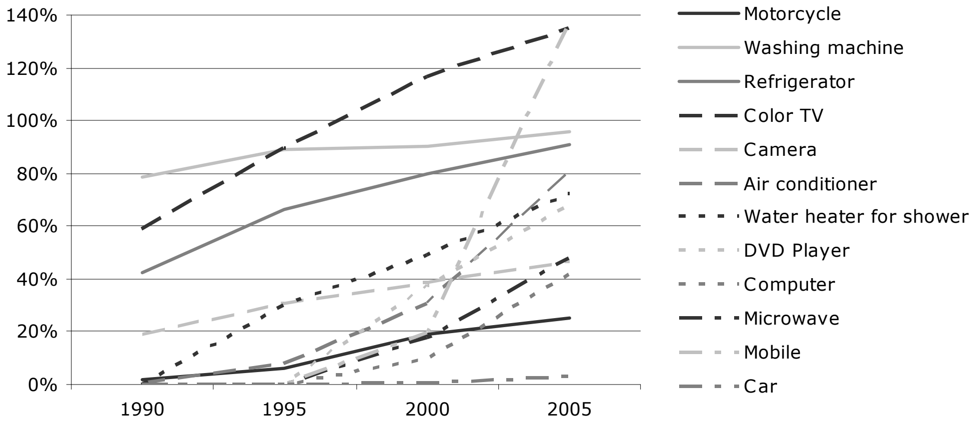
$$\begin{aligned}
\frac{L}{F}\left[p_e^1-(1-\beta)a_h\right]-\rho &< \frac{L\left[\left(\psi+(1-\psi)m(p_e^1)^\gamma\right)^{1/\gamma}-(1-\beta)a_h-\beta a_l m(p_e^1)\right]}{F+Gm(p_e^1)} \\
\frac{L}{F}\left[p_e^2-(1-\beta)a_h\right]-\rho &> \frac{L\left[\left(\psi+(1-\psi)m(p_e^2)^\gamma\right)^{1/\gamma}-(1-\beta)a_h-\beta a_l m(p_e^2)\right]}{F+Gm(p_e^2)}
\end{aligned} \tag{13}$$

and $p_e^2 > p_e^1$. By means of an example it is easy to show that there are parameter values that satisfy (13).

Computations have shown that Assumption 1 holds in a balanced growth equilibrium if the quality gap $q_h - q_l$ is sufficiently high relative to the cost gap $a_h - a_l$ and process innovation costs G; and if inequality is sufficiently high, i.e. the group of poor β is sufficiently large as well as θ not too high. Notice further that $a_l\beta/(1-\beta) > a_h G/F$ (positive slope of N-curve) must hold to ensure a separating equilibrium. The reason is that $a_l\beta/(1-\beta) \le a_h G/F$ necessarily goes together with $p_e^2 \le p_e^1$.

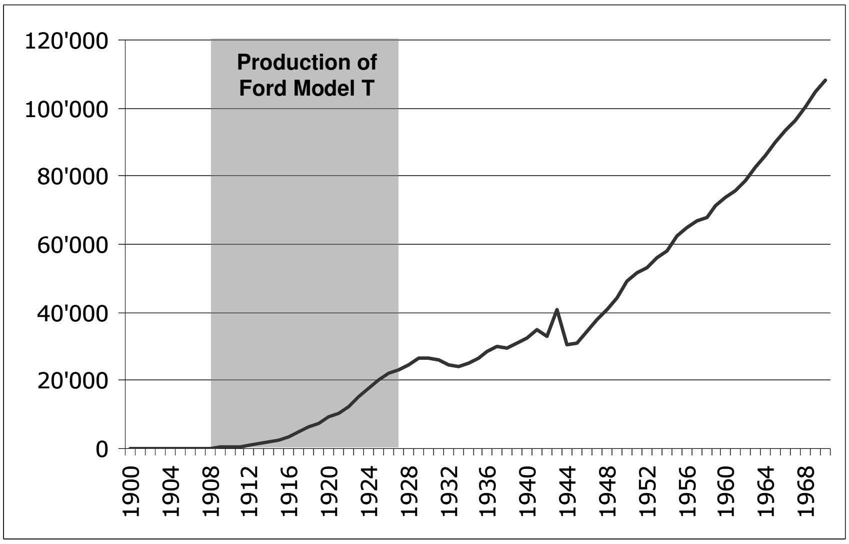
If condition (13) is violated, a positive balanced growth equilibrium may still exist but it is not necessarily unique. Apart from two locally stable steady states (one stagnatory) there exists an intermediate unstable steady state in that case.

Figure 1: Ownership of consumer durables in Urban Chinese households



Source: National Bureau of Statistics of China.

Figure 2: Automobile and truck registrations in the USA in 1,000 units



Source: US Census.

Figure 3: The LORENZ curve

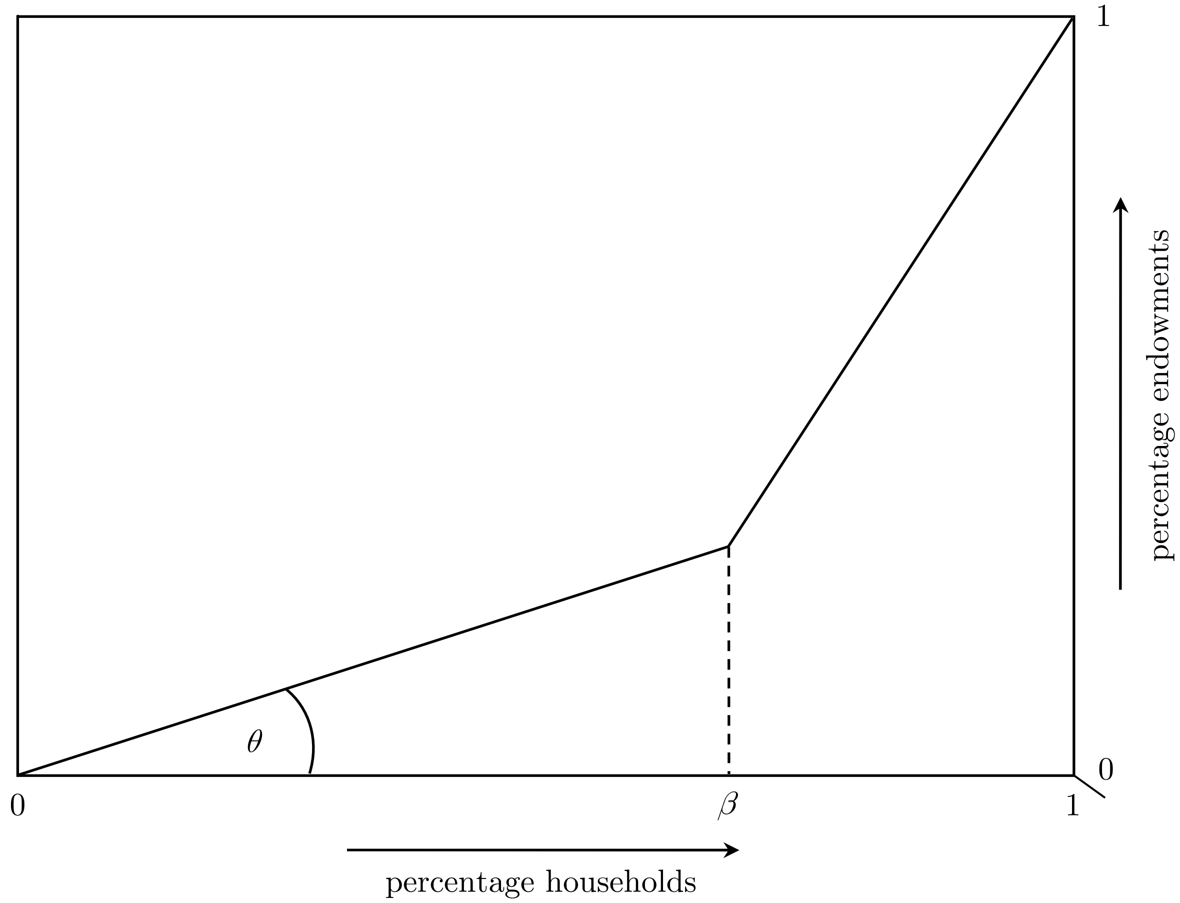
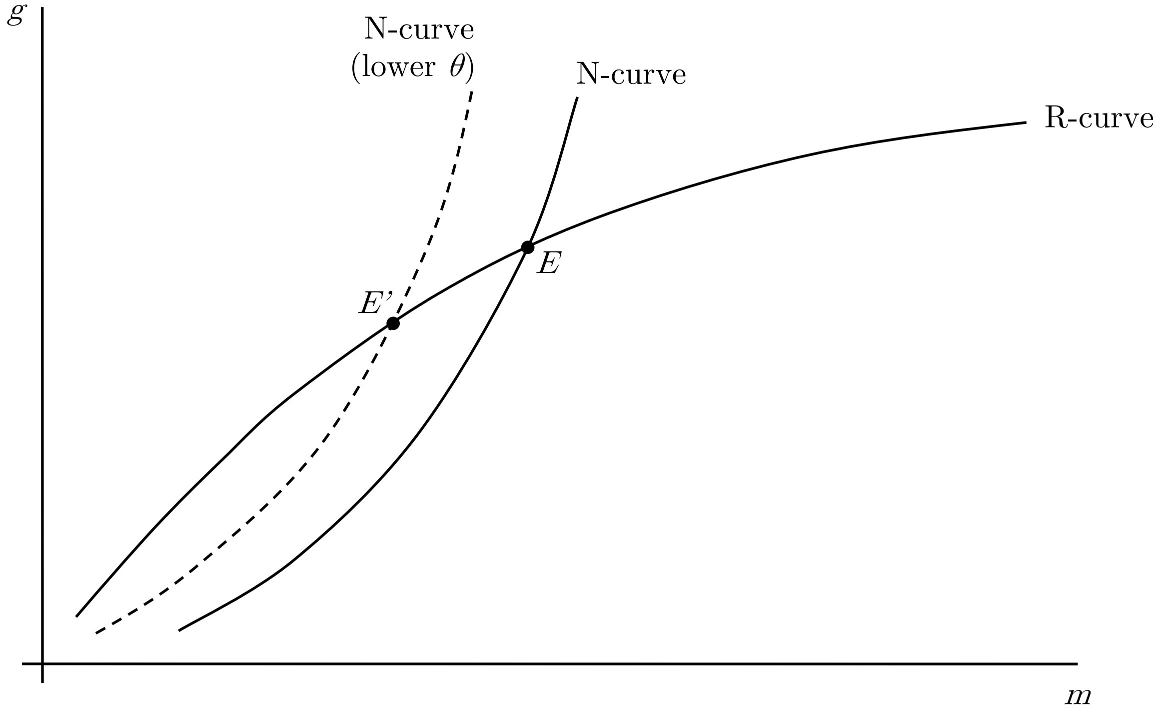
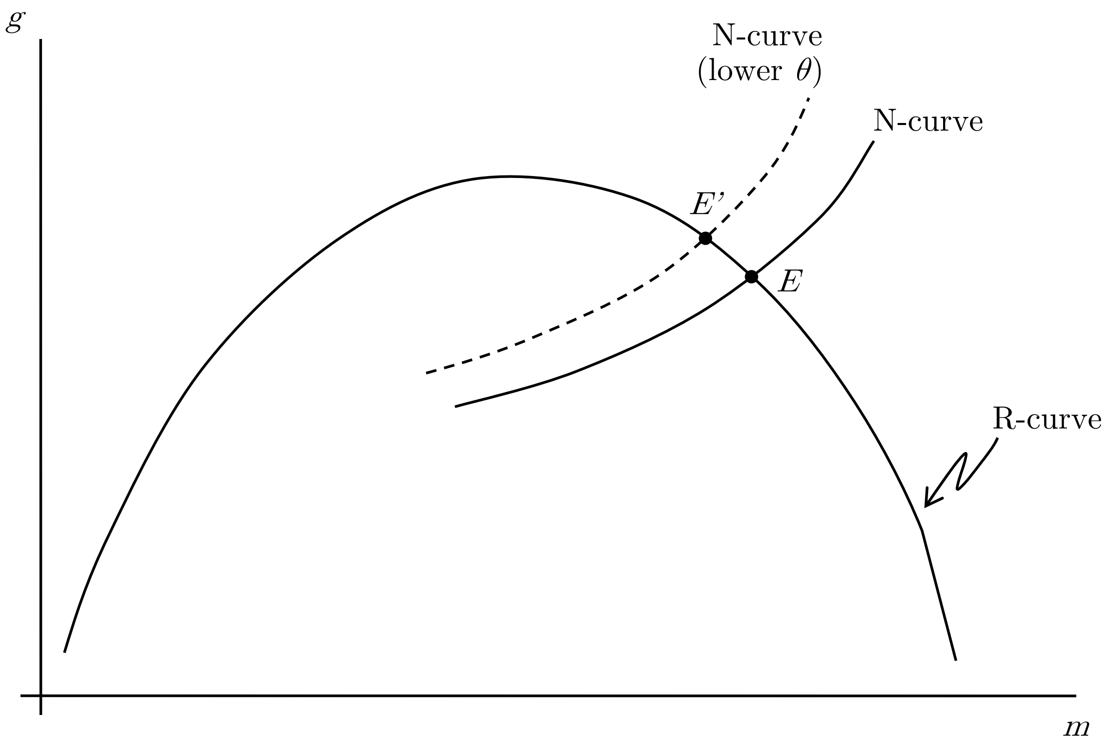


Figure 4: A larger income gap (lower θ)



(a)



(b)

Figure 5: Higher income concentration (larger β)

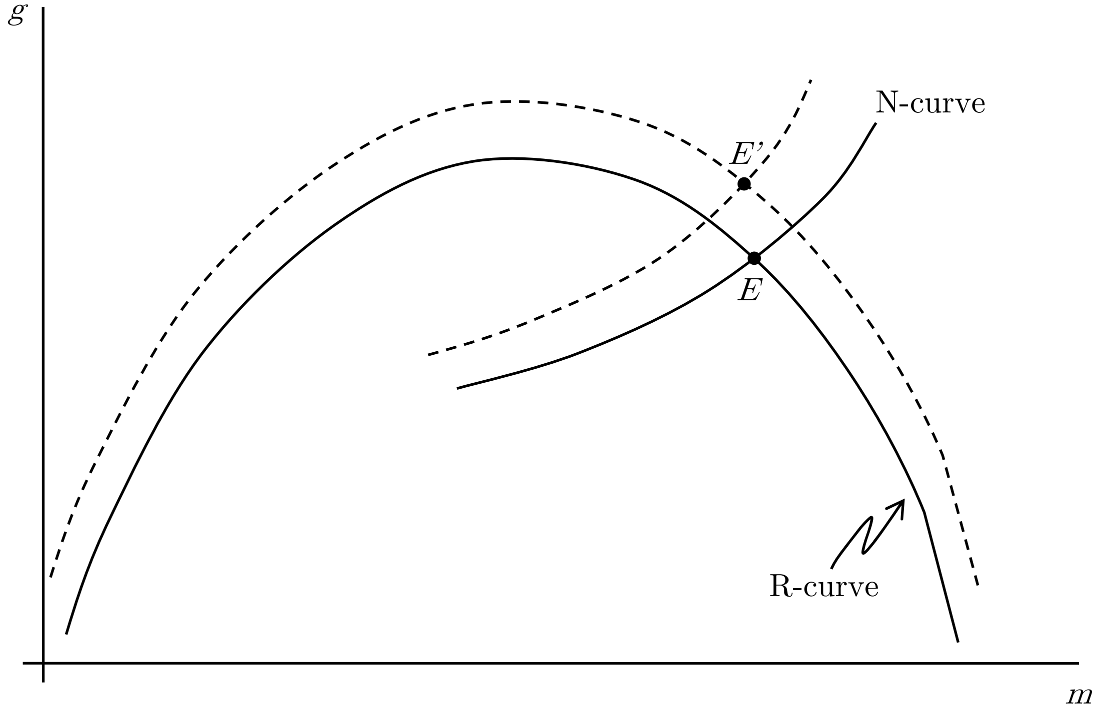
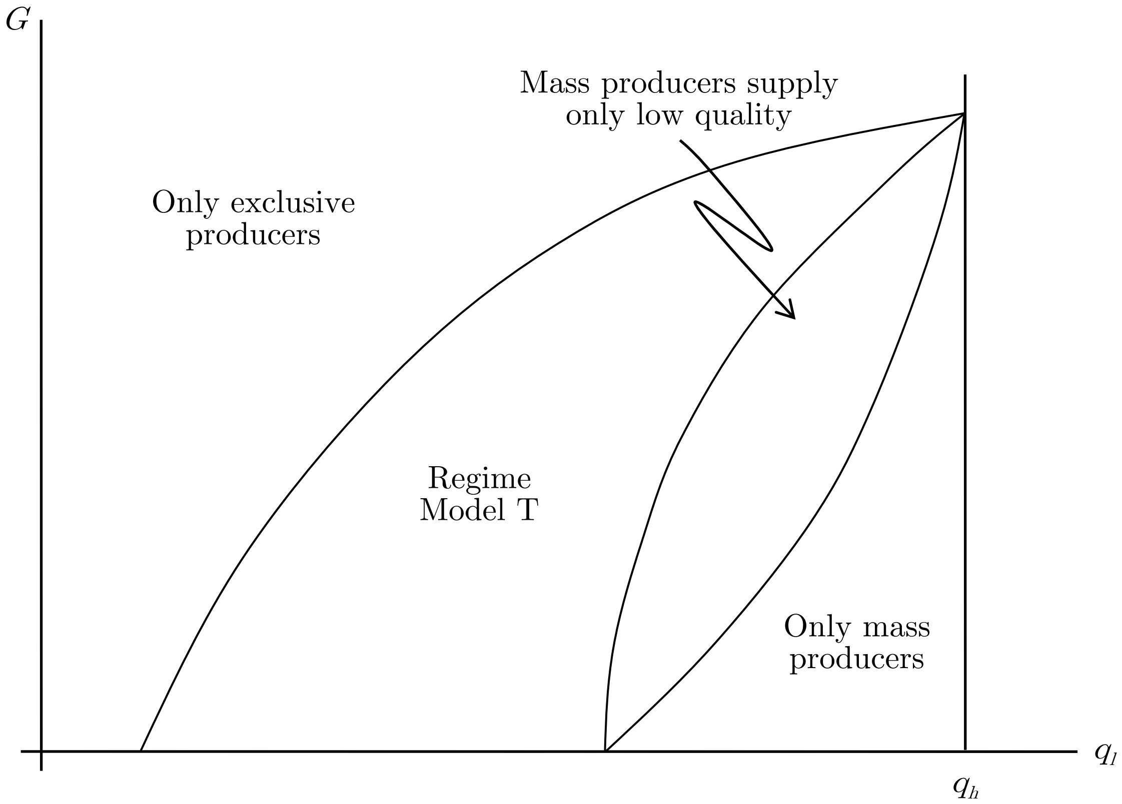


Figure 6: Alternative balanced growth regimes



Online Appendix "Transitional Dynamics" to: THE MACROECONOMICS OF MODEL T

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The general equilibrium of our model has two state variables: the number of mass producers $M(t)$ and the total number of firms $N(t)$. What happens if the ratio $M(t)/N(t)$ deviates from its balanced-growth level? To illustrate the transition process, consider an economy in steady state and assume that this economy experiences a major redistribution from rich to poor. In section 4 we have seen that a major drop in inequality leads to a new balanced growth path with a higher extent of mass production and, provided that inequality is initially high, most likely also to higher growth. The substantial drop in inequality during the Great Depression and WWII and the boom in consumer durables in the post-war era provides a potentially relevant example from recent economic history. Technically, we assume throughout that the redistribution is major but in a size that the new steady state is again a separating equilibrium. It turns out that, when the economy operates along the balanced growth path both variables grow pari passu. When the economy operates off this path, there are either only product innovations or only process innovations but not both. We summarize this result in

Proposition 1 *Suppose Assumption 1 holds and the economy features both product and process innovations. Then the economy is on the balanced growth path.*

Proof. *Suppose the economy is in an equilibrium but not necessary the steady state where both product and process innovation occur. Since $V_N(t) = F$ and $V_M(t) = G$ hold, the instanta-*

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neous interest rate is given by

$$r(t) = L\left[q_l\mu_P(t) - (1-\beta)q_l\mu_R(t) - \beta a_l\right]/G = (1-\beta)L(q_h\mu_R(t) - a_h)/F.$$

The Euler equations of rich and poor, and the resource constraint read

$$\begin{aligned} \dot{\mu}_R(t)/\mu_R(t) &= r(t) - \rho - \dot{N}(t)/N(t), \quad \dot{\mu}_P(t)/\mu_P(t) = r(t) - \rho - \dot{M}(t)/M(t), \\ \dot{M}(t)G + \dot{N}(t)F &= L\left(\psi N(t)^\gamma + (1-\psi)M(t)^\gamma\right)^{1/\gamma} - L\beta M(t)a_l - L(1-\beta)N(t)a_h. \end{aligned}$$

We reduce this system of differential equations to get a single equation in $\mu_R(t)$ and $M(t)/N(t)$. Rewrite the resource constraint

$$\frac{\dot{M}(t)}{M(t)}\frac{M(t)}{N(t)}G + \frac{\dot{N}(t)}{N(t)}F = L\left(\psi + (1-\psi)\left(\frac{M(t)}{N(t)}\right)^\gamma\right)^{1/\gamma} - L\beta\frac{M(t)}{N(t)}a_l - L(1-\beta)a_h \equiv \phi\left(\frac{M(t)}{N(t)}\right)$$

Rearranging the above equation for $r(t)$ we get $q_l\mu_P(t) = (1-\beta)\left(q_l + q_hG/F\right)\mu_R(t) + \beta a_l - (1-\beta)a_hG/F$. We take the derivative and insert this into the Euler equation of the poor to get

$$\frac{\dot{\mu}_R(t)}{\mu_R(t) + \left[\beta a_l - (1-\beta)a_hG/F\right]/\left[(1-\beta)\left(q_l + q_hG/F\right)\right]} =$$
$$(1-\beta)\frac{L}{F}(q_h\mu_R(t) - a_h) - \rho - \left(\frac{M(t)}{N(t)}G\right)^{-1}\left(\phi\left(\frac{M(t)}{N(t)}\right) - \frac{\dot{N}(t)}{N(t)}F\right),$$

and use the Euler equation of the rich to form

$$\frac{\dot{\mu}_R(t)}{\mu_R(t) + \left[\beta a_l - (1-\beta)a_hG/F\right]/\left[(1-\beta)\left(q_l + q_hG/F\right)\right]} + \frac{\dot{\mu}_R(t)}{\mu_R(t)}\frac{F}{GM(t)/N(t)} =$$
$$(1-\beta)\frac{L}{F}(q_h\mu_R(t) - a_h) - \rho - \left(\frac{M(t)}{N(t)}G\right)^{-1}\left(\phi\left(\frac{M(t)}{N(t)}\right) - (1-\beta)L(q_h\mu_R(t) - a_h) + \rho F\right).$$

We see that $\dot{\mu}_R(t)$ is monotonically increasing in $\mu_R(t)$. Denote the steady state level of $\mu_R(t)$ by μ_R^{SS}. Therefore, if $\mu_R(t) > (<)\mu_R^{SS}$, $\mu_R(t)$ will grow (fall) without bound. Hence, there is only one equilibrium: $\mu_R(t)$ must immediately adjust to μ_R^{SS}. As $\mu_P(t)$ and $\mu_R(t)$ are monotonically related, the analoguous holds true for $\mu_P(t)$ as well. We conclude that in the presence of both process and product innovations the economy is in steady state. ■

The proposition implies that, when the economy has too few mass producers $M(t)$, the transition process will be characterized by process innovations only. Similarly, if there are too few exclusive producers $N(t) - M(t)$, the transition process will be characterized only by product innovations. Hence, all adjustments in the state variable $m(t) = M(t)/N(t)$ occur by a "bang-bang" rule. We will also see that this implies that the transition from an old to a new steady state will occur in finite time. This is partly driven by the assumption that $A(t)$ is common across product and process innovation so that the relative cost of the two types of innovations never change. A phase in which one engine of growth stops temporarily is not specific to our set-up.[1]

In what follows we consider the transition process triggered by two alternative parameter changes (i) a major drop in inequality; and (ii) a drastic reduction in the cost for process innovations. We assume that both in the initial and final balanced growth equilibrium conditions are such that exclusive producers sell (their high quality) only to the rich; and mass producers sell the high quality to rich and the low quality to poor households. In contrast to the analysis of the last section, we need to relax the assumption of identical endowment distributions. This is because the transition process will be characterized by a situation where the two types of households face different incentives to save and hence will accumulate wealth at unequal speed. In other words, in the transition process, the wealth distribution is no longer stationary invalidating the assumption $\theta_\ell = \theta_v = \theta$. Instead we need to account for the fact that $\theta_v(t)$ changes over time. We focus on the case of log-utility, $\sigma = 1$, for simplicity.

The initial and final balanced growth paths are still characterized by the same R-curve as in the main text. However, the N-curve has to be adjusted. This is because θ_ℓ may no longer be equal to θ_v, as prices for mass and exclusive products (as well as the percentage of mass and exclusive producers) change during transition. With a constant interest rate r and a constant growth rate g, the present value of household i's lifetime income (the right-hand-side of the household i's intertemporal budget constraint) equals $w(t)\ell_i/\rho + v_i(t)$. By normalization, the wage is equal to $w(t) = A(t) = N(t)\,(\psi + (1-\psi)m^\gamma)^{1/\gamma}$ and, from the zero-profit condition, we have $v(t)L = N(t)\,(F + mG)$. As the left-hand-side of a household's intertemporal budget constraint is unaffected by the more general specification of the endowment distributions, we can rewrite the relative budget constraints of the two types of consumers as

$$\frac{mp_h + (1-m)p_e}{mp_l} = \xi(m),$$

where relative lifetime incomes $\xi(m)$ are now given by

$$\xi(m) \equiv \frac{\rho(1-\beta\theta_v)(F+mG) + (1-\beta\theta_\ell)L\,(\psi + (1-\psi)m^\gamma)^{1/\gamma}}{\rho(1-\beta)\theta_v(F+mG) + (1-\beta)\theta_\ell L\,(\psi + (1-\psi)m^\gamma)^{1/\gamma}},$$

with $\xi_m(m) > 0$ since $(\theta_v, \theta_\ell) < (1,1)$. Note also that $\xi(m)$ decreases in both θ_v and θ_ℓ.

We can solve this more general case in a similar way as above. First calculate $p_e = q_h\mu_R$ using equation the consumers' budget constraints and the no-arbitrage conditions. Then plug

[1] See Matsuyama (1999) for another example where in one phase product variety expansion stops, while the economy accumulates physical capital. In our framework, expansion of variety stops while the economy accumulates process innovation. In fact, this transition closely resembles the related work of directed technical change (see Proposition 1 of Acemoglu and Zilibotti, 2001), where only one type of innovation takes place outside the balanced growth equilibrium.

the resulting expression into the no-arbitrage condition for the exclusive producer to get a new no-arbitrage curve

$$g = L(1-\beta)\frac{\hat{p}(m) - a_h}{F} - \rho,$$

where, to save notation, we have set $\sigma = 1$. In this more general framework, where firm shares and labor endowments are not identically distributed, the price for exclusive goods, $\hat{p}(m)$ is given by

$$\hat{p}(m) = q^h \frac{\beta a_l - (1-\beta) a_h G/F}{(q_h/m - q_l)\,/\,(\xi(m) - 1) - (1-\beta)\,(q_l + q_h G/F)}.$$

Similar to the case with identical distributions, raising θ_v or θ_ℓ increases m, since $\xi(m)$ is increasing in m as well as decreasing in θ_v and θ_ℓ. Lowering financial wealth or labor income inequality reduces exclusion. Hence, in much the same way as above, the inequality-growth relationship depends on the slope of the resource curve.

Assume there is a major drop in inequality, i.e. a mean-preserving spread in the endowment distributions raising incomes of poor households at the expense of the rich, so that $(\theta_v'(t_0), \theta_\ell') > (\theta_v, \theta_\ell)$ at time $t = t_0$. The following proposition characterizes the transition process.

Proposition 2 *Suppose Assumption 1 holds at all times. a) A fall in inequality at date t_0, from (θ_v, θ_ℓ) to $(\theta_v'(t_0), \theta_\ell')$, triggers a transition period of finite duration (t_0, t_2) where $\dot{N}(t) = 0$ and $\dot{M}(t) > 0$. A new balanced growth equilibrium with $m' > m$ is reached at date t_2. b) During the entire transition period consumption of the rich stagnates at $c_R(t) = q_h N(t_0)$. c) When the initial reduction in inequality is substantial, $c_P(t)$ jumps to a higher level at date t_0. During a first transition period, $t \in [t_0, t_1)$, $c_P(t) > q_l M(t)$; during a second transition period, $t \in [t_1, t_2)$, $c_P(t) = q_l M(t)$. When the initial reduction in inequality is minor, $c_P(t)$ does not change discontinuously at date t_0, the first transition period does not exist and $c_P(t) = q_l M(t)$ for all $t > t_0$.*

On impact, the consumption level of the poor jumps to $N_P(t_0) > M(t_0)$ (they consume also high-quality goods). The transition process is characterized by two phases during which only process innovations and no product innovations take place. During the *first phase* (t_0, t_1) the laws of motion are

$$\begin{aligned}
\dot{\mu}_R(t)/\mu_R(t) &= r_1 - \rho, \\
\dot{\mu}_P(t)/\mu_P(t) &= r_1 - \rho - \left[\dot{M}(t) q_l + \left(\dot{N}_P(t) - \dot{M}(t)\right) q_h\right] / \left[M(t) q_l + (N_P(t) - M(t))\, q_h\right], \\
\dot{M}(t) G/L &= A(N(t_0), M(t)) - \beta\left[M(t) a_l + (N_P(t) - M(t)) a_h\right] - (1-\beta) N(t_0) a_h,
\end{aligned}$$

and $\dot{N}(t) = 0$, with $r_1 = [q_l/q_h - a_l/a_h]\, L\beta a_h/G$, and $\mu_P(t) = (1-\beta)\mu_R(t) + \beta a_h/q_h$. Moreover, we have initial values for the state variables, $N(t_0) > 0$, and $M(t_0) \geq 0$. The equal-profit and transversality conditions for rich and poor households fix initial values for the costate variables, $\mu_R(t_0)$ and $\mu_P(t_0)$ (and $N_P(t_0)$). Numerically, we solve the system by backward integration starting in the final balanced growth equilibrium and letting time run backward. Initial values of costate variables thus can be fixed by using final balanced growth equilibrium values as boundary conditions. Since prices of mass and exclusive goods evolve differently, wealth inequality, $\theta_v(t)$, changes during the transition,

$$\begin{aligned}
\dot{v}_R(t) &= r_1 v_R(t) + (1-\beta\theta_\ell)/(1-\beta)A(N(t_0), M(t)) - [N(t_0) - N_P(t)]\, q_h\mu_R(t) - \\
&\quad [N_P(t) - M(t)]\, q_h\mu_P(t) - M(t)\,[(q_h - q_l)\mu_R(t) + q_l\mu_P(t)]\,, \\
\dot{v}_P(t) &= r_1 v_P(t) + \theta_\ell A(N(t_0), M(t)) - [N_P(t) - M(t)]\, q_h\mu_P(t) - M(t)q_l\mu_P(t).
\end{aligned}$$

Initial wealth inequality can be fixed using final values as boundary conditions if we know final wealth inequality, θ_v'. If we know initial wealth inequality instead, $\theta_v(t_0)$, we guess final wealth inequality, shoot backward, and check whether the resulting initial wealth inequality corresponds to the true value. This process is reiterated with new guesses until a sufficiently close value is found (see below).

The economy enters the *second phase* (t_1, t_2) as soon as $N_P(t) = M(t)$. The laws of motion are

$$\begin{aligned}
\dot{\mu}_R(t)/\mu_R(t) &= r(t) - \rho, \\
\dot{\mu}_P(t)/\mu_P(t) &= r(t) - \rho - \dot{M}(t)/M(t), \\
\dot{M}(t)G/L &= A(N(t_0), M(t)) - \beta M(t)a_l - (1-\beta)N(t_0)a_h,
\end{aligned}$$

and $\dot{N}(t) = 0$, with $r(t) = [q_l\mu_P(t) - (1-\beta)q_l\mu_R(t) - \beta a_l]\, L/G$. Since all mass producers have innovated, the equal profits equation does not need to hold anymore, and interest rates are no longer constant. Initial values of state variables are given by the values at the end of phase 1, $N(t_0)$ and $M(t_1)$. Final conditions using backward integration fix the level of costate variables, $\mu_R(t)$ and $\mu_P(t)$. Wealth accumulation is

$$\begin{aligned}
\dot{v}_R(t) &= r(t)v_R(t) + (1-\beta\theta_\ell)/(1-\beta)A(N(t_0), M(t)) - \\
&\quad [N(t_0) - M(t)]\, q_h\mu_R(t) - M(t)\,[(q_h - q_l)\mu_R(t) + q_l\mu_P(t)]\,, \\
\dot{v}_P(t) &= r(t)v_P(t) + \theta_\ell A(N(t_0), M(t)) - M(t)q_l\mu_P(t).
\end{aligned}$$

The economy exits phase 2 once product innovation becomes attractive again, $r(t)F = (1-\beta)L(q_h\mu_R(t) - a_h)$, and enters the new balanced growth equilibrium (given Proposition 4). Note that an economy never skips phase 2 in a transition to a higher m', directly entering the new balanced growth path after phase 1 (i.e. every such transition contains phase 2). Since in phase

1, exclusive producers make equal profits selling only to rich or to all households, and this is not the case in the final steady state (given Assumption 1 with strict inequalities), there needs to be a phase where prices adjust accordingly (as costate variables cannot jump expectedly).[2]

Notice also that during the two phases of transition the interest rates is constant and given by[3]

$$r_1 G = \left[\frac{q_l}{q_h} - \frac{a_l}{a_h}\right] L\beta a_h.$$

Moreover, during the first phase an (endogenous) fraction of exclusive producers sells also to poor households. During that period, exclusive producers are indifferent between selling to all households and selling only to the rich

$$L\left(q_h\mu_P(t) - a_h\right) = L(1-\beta)\left(q_h\mu_R(t) - a_h\right).$$

[2]Finally, transitional dynamics and numerical simulations allow us to analyze the stability of the balanced growth equilibrium of Section 5. For most parameter values, the equilibrium is globally saddle path stable. If the economy starts with a too low m, we enter a transitional phase characterized above with no product innovation and only process innovation. In contrast, if the economy starts with a too high m, *mutatis mutandis*, society goes through a phase without process innovations and only product innovations, reaching the balanced growth equilibrium in finite time (with an initial phase where mass and exclusive producers earn equal profit flows, and some mass producers do not use the mass production process and only sell the high quality to rich households). The numerical simulation procedure uses backward integration (Brunner and Strulik, 2002) to tackle transitional dynamics, analyzing the dynamic system numerically with the Mathematica procedure "NDSolve". However, since transition is finite and has different phases, we need to make adjustments to the standard procedure. The key steps in this procedure are: (1) We start by solving the final and the initial balanced growth equilibrium. (2) Using the differential equations derived above, we let time run backward by multiplying the right-hand side of the ordinary differential equation system with the scalar (-1). (3) Hence, we start in phase 2, solve for the path of state and costate variables, then solve phase 1, using "NDSolve". (4) To determine at what point the economy switches to the preceding phase we keep track of the no-arbitrage conditions. As an example, going backward in phase 2, as soon as the mass high strategy becomes attractive, we have reached the start of phase 2, and thus the end of phase 1. The values of state/co-state variables at the calculated point of time serve as ending values for the preceding phase. (5) If we are in phase 1, time running backwards, as soon as $m(t)$ hits the initial value, we know that we are at the time of the shock, t_0. (6) Having programed all phases, we need to take the final balanced growth equilibrium state/co-state variables and let time run backward. Note that since in our model the transition period is finite, we do not need to perturb final balanced growth path values slightly as would be the case in the standard procedure if convergence were asymptotic. We simply need to start with the dynamic system of phase 2 using the exact values of the final balanced growth path variables. (7) If we know final wealth inequality θ_v', we can track wealth levels backward (using the wealth accumulation equations), computing θ_v at time t_0. If we know initial wealth inequality $\theta_v'(t_0)$ instead, we must guess final wealth inequality, shoot backward, and check whether the resulting initial wealth inequality corresponds to the true value. This process must be reiterated with new guesses until one is sufficiently close to the true value.

[3]We use the equal-profit condition to eliminate the willingness-to-pay of rich and poor in the incremental profit flow, $L\left(q_l\mu_P(t) - (1-\beta)q_l\mu_R(t) - \beta a_l\right)$. The flow must be equal to $r_1 G$ since $V_P(t) = G$ and thus $\dot{V}_P(t) = 0$.

The Euler equation determines the growth rate of the willingness to pay of the rich and the poor

$$\frac{\dot{\mu}_R(t)}{\mu_R(t)} = r_1 - \rho, \quad \text{and} \quad \frac{\dot{\mu}_P(t)}{\mu_P(t)} + \frac{\dot{c}_P(t)}{c_P(t)} = r_1 - \rho.$$

Since mass producers and exclusive producers earn the same profits during the first transition phase, it must be that $\mu_P(t)$ increases at a smaller rate than $r_1 - \rho$.[4] Consequently, $\dot{c}_P(t)/c_P(t) > 0$. Denote by $N_P(t)$ the number of goods that the poor can afford. During the first period of transition we have $N_P(t) > M(t)$ and $c_P(t) = q_l M(t) + q_h(N_P(t) - M(t))$. Since $M(t)$ grows faster than $N_P(t)$, there is a date $t = t_1$ where we have reached $M(t_1) = N_P(t_1)$. From date t_1 onwards we have $c_P(t) = q_l M(t)$. The equal-profit condition does not hold anymore and exclusive producers are strictly better off selling only to the rich. $\mu_R(t)$ continues to grow at rate $r(t) - \rho$, but $\mu_P(t)$ grows more slowly. Interest rates are no longer constant, but still determined by incremental profit flows and investment costs for process innovation. The final law of motion comes from the resource constraint. Recalling that in the entire transition period we have $\dot{N}(t) = 0$ and $N(t) = N(t_0)$ we can write

$$\dot{M}(t)G/L = A(N(t_0), M(t)) - \beta\left[M(t)a_l + (N_P(t) - M(t))a_h\right] - (1-\beta)N(t_0)a_h.$$

Moreover, we have initial conditions $M(t_0) = mN(t_0)$ and $N(t_0)$, and transversality conditions for rich and poor households. At date t_2, the economy reaches the new balanced growth equilibrium with $m(t) = m'$ in finite time as soon as product innovation becomes attractive again, $r(t)F = L(1-\beta)(q_h\mu_R(t) - a_h)$.

Figure 7 summarizes the dynamics of consumption paths of rich and poor households. The redistribution occurs at date t_0 and reduces the willingness to pay of the rich and increases the one of the poor households. As a result, process innovations become strictly more attractive than product innovation and, during the entire transition process, only process R&D is undertaken.[5] As no additional products are invented, consumption by the rich stagnates. Consumption by the poor immediately jumps to a higher level. They become siddenly rich enough to consume also high-quality goods. The first phase of transition is characterized by a situation where more and more mass goods are brought to the market and the poor continuously replace old high-quality by new low-quality items. The first phase ends at date t_1, when the poor start to consume again only low-quality goods. In the second phase consumption of the poor grows pari passu with

[4]Because of the equal-profit condition it is straighforward to calculate $\dot{\mu}_P(t)/\mu_P(t) = [(1-\beta)\dot{\mu}_R(t) + \beta a_h] / [(1-\beta)\mu_R(t) + \beta a_h] < \dot{\mu}_R(t)/\mu_R(t) = r_1 - \rho$

[5]When the economy has too few mass producers $M(t)$, the transition process will be characterized by process innovations only. Similarly, if there are too few exclusive producers $N(t) - M(t)$, the transition process will be characterized only by product innovations. Hence, all adjustments in the state variable $m(t) = M(t)/N(t)$ occur by a "bang-bang" rule.

mass produced goods $M(t)$ but process R&D remains the only R&D activity. The second phase of transition ends at date t_2 when the new balanced growth ratio m is reached.

FIGURE ONLINE APPENDIX

It is interesting to look at firms' price setting behavior during transition. During the *first* phase, exclusive producers (who can supply only high quality) are confronted with a higher willingness to pay by the poor. An (endogenous) fraction of these producers will therefore set a price that equals the willingness to pay of the poor and sell temporarily to all households; and the remaining fraction of exclusive producers continues to sell only to the rich at a price equal to their (high) willingness to pay. Over the time intervall (t_0, t_1) the fraction of exclusive producers selling to all households falls continuously to zero.

During the *second* transition phase (t_1, t_2) all exclusive and mass producers sell their high quality to the rich and all mass producers sell their low quality to the poor. However, during the second transition period, the willingness to pay by the rich increases more strongly than the willingness to pay by the poor. While the incomes of both types of households increase due to technical progress, only the poor can expand consumption while the rich are constrained by the (unchanged) number of high-quality goods. In other words, the willingness to pay of the rich rises faster than the one of the poor. By date t_2, when the new balanced growth path is reached, the willingness to pay of the rich has increased so much that product innovations become again as attractive as process innovations.

References

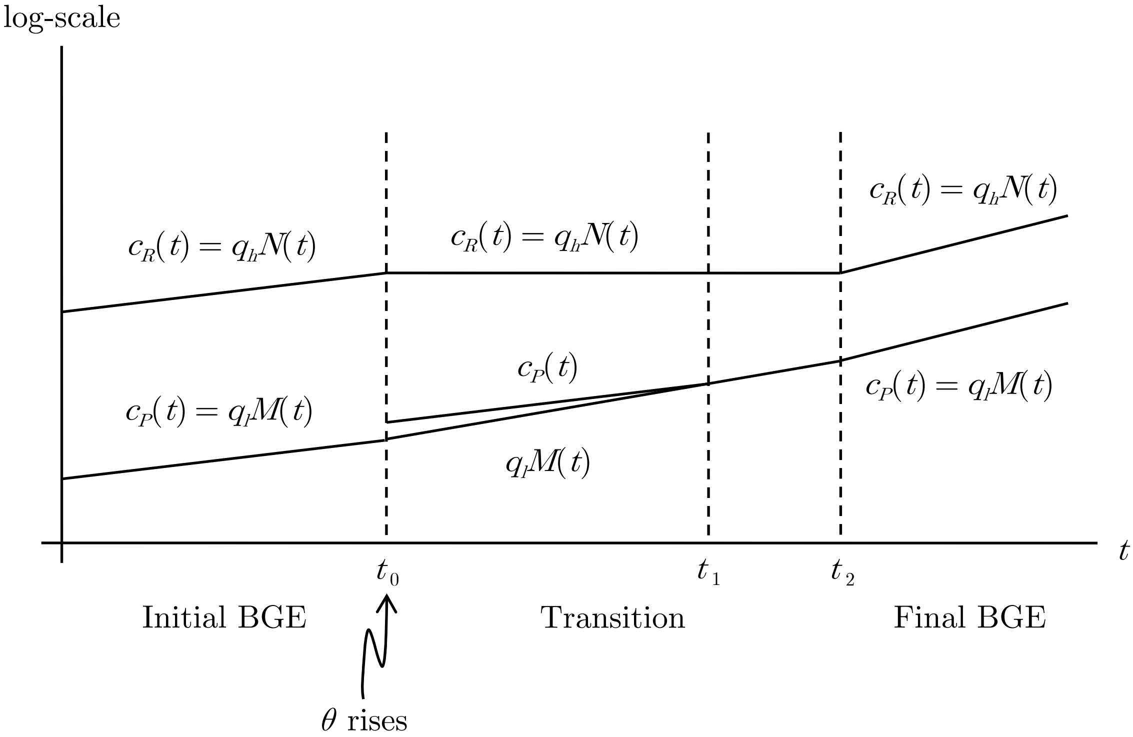
[1] Acemoglu, Daron and Fabrizio Zilibotti (2001), "Productivity Differences," *Quarterly Journal of Economics* 116, 563-606.

[2] Galor, Oded and Omer Moav (2004), "From Physical to Human Capital Accumulation: Inequality and the Process of Development," *Review of Economic Studies* 71, 1001-1026.

[3] Galor, Oded and Omer Moav (2006), "Das Human Kapital: A Theory of the Demise of the Class Structure," *Review of Economics Studies* 73, 85-117.

[4] Matsuyama, Kiminori (1999), "Growing Through Cylces," *Econometrica* 67, 335-347.

Figure 1: Transitional dynamics (fall in inequality, larger θ)



$q_l M(t)$... Consumption of mass goods of poor households

$c_P(t) = q_l M(t) + q_h[N_P(t) - M(t)]$... Consumption of poor households

$c_R(t) = q_h N(t)$... Consumption of rich households